

# Numerical Weather Prediction

ATS703 is based on the course notes and papers available over the web at the Schubert Research Group Web Site under “Teaching,” with the user ID and password given in class. You can print out individual chapters from the website. The complete list of chapters is given below.

The course begins with a review of the equations for a dry atmosphere (Chapter 1). These governing equations are then written in spherical coordinates (Chapter 2), after which the traditional approximation and the quasi-hydrostatic approximation are discussed (Chapters 3 and 4). The quasi-hydrostatic approximation allows a wide variety of vertical coordinates to be used, and these are presented in Chapter 5. Chapters 6–9 discuss the spherical harmonic spectral method, which has been so wonderfully used by ECMWF. Chapter 10 discusses the semi-Lagrangian method, an ingenious method that can be paired with the spectral method. A crucial element of accurate weather prediction is initialization, which is briefly discussed in Chapter 11. In the next decade, numerical weather prediction will experience a revolution in model formulation as we transition from quasi-hydrostatic models to non-hydrostatic models. In preparation for this, the last decade has seen considerable effort to formulate global non-hydrostatic models with fine enough resolution that they can be considered “cloud permitting” or even “cloud resolving,” thereby eliminating the need for cumulus parameterization. Some of the issues involved are discussed in Chapters 12–14. These efforts are continuing at many institutions, for example: (1) the National Centers for Environmental Prediction (NCEP); (2) the National Center for Atmospheric Research (NCAR) and the Los Alamos National Laboratory (LANL) with the Model for Prediction Across Scales (MPAS); (3) Deutscher Wetterdienst and the Max Planck Institute for Meteorology with the ICON model; (4) Met Office with the non-hydrostatic Unified Model (UM) and the more recent New Dynamics Model (ND); (5) Meteo-France development of the non-hydrostatic NWP model AROME; (6) Non-hydrostatic Icosahedral Atmospheric Model (NICAM, Tomita et al. 2010); (7) Evolution of the European Center for Medium Range Weather Forecasts (ECMWF) Integrated Forecast System (IFS) to IFS/ARPEGE/ALADIN/AROME. The models differ in many details. They avoid the pole problem of latitude/longitude grids by generating the grid from the icosahedron. For example, MPAS uses a variable resolution Voronoi mesh while the NICAM uses a unique mesh based on “spring dynamics.” They also differ in the details of how sound waves are treated, i.e., on whether the basic equations are soundproof or fully compressible. Unfortunately, there is little agreement on the treatment of moist thermodynamics. An excellent introduction to this literature can be found in the Proceedings of the ECMWF Workshop on Non-Hydrostatic Modeling, where many important references can also be found.

1. Introduction
  - 1.1 Ideal gas law
  - 1.2 Material derivative
  - 1.3 Mass conservation
  - 1.4 Thermodynamic equation
  - 1.5 Noninertial reference frames
  - 1.6 The Foucault pendulum
  - 1.7 The complete system of equations for a dry atmosphere
2. The Exact Primitive Equations
  - 2.1 Exact primitive equations in spherical coordinates
  - 2.2 Summary
3. Primitive Equations for Shallow Atmospheres
  - 3.1 Primitive equations with the traditional approximation
  - 3.2 Summary
4. The Quasi-hydrostatic Primitive Equations
  - 4.1 Scale analysis

- 4.2 Summary
5. Vertical Coordinates Used in Numerical Weather Prediction
  - 5.1 The general  $\eta(p, p_s, \theta)$  coordinate
  - 5.2 Pressure coordinate
  - 5.3 Log-pressure coordinate
  - 5.4 Pseudo-height coordinate
  - 5.5 Sigma coordinate
  - 5.6 Isentropic coordinate
  - 5.7 The ECMWF hybrid vertical coordinate
  - 5.8 Arakawa-Konor hybrid  $\sigma$ - $p$  vertical coordinate
  - 5.9 Konor-Arakawa hybrid  $\theta$ - $p$ - $p_S$  vertical coordinate
  - 5.10 Hydrostatic pressure as a vertical coordinate in non-hydrostatic models
6. Spherical Harmonic Spectral Models
  - 6.1 The eigenfunctions of the Laplacian operator on the sphere
  - 6.2 Representation of the earth's topography using spherical harmonics
  - 6.3 Spatial smoothing on the sphere
7. Nondivergent Barotropic Model
  - 7.1 Linear Rossby-Haurwitz waves
  - 7.2 Spectral method
  - 7.3 Pseudospectral method
  - 7.4 Energy dispersion in a barotropic atmosphere
  - 7.5 Rossby-Haurwitz wave breaking
  - 7.6 Two-dimensional turbulence on the sphere
8. Shallow Water Model
  - 8.1 Vorticity/divergence form
  - 8.2 Spectral transform method
  - 8.3 Semi-implicit time differencing
9. Quasi-Static Primitive Equation Models on the Sphere
  - 9.1 Mathematical formulation
  - 9.2 Baroclinic instability on the sphere
  - 9.3 Barotropic influences on the growth and decay of nonlinear baroclinic waves
  - 9.4 The ECMWF global model
10. Semi-Lagrangian Methods (Staniforth and Cote 1991)
  - 10.1 Passive advection in one and two dimensions
  - 10.2 Three-time-level and two-time-level advection methods
  - 10.3 Pairing semi-Lagrangian and spectral methods (Ritchie 1988, 1991)
  - 10.4 Climate simulations with a spectral, semi-Lagrangian model with linear grids (Williamson 1995)
11. Initialization

- 11.1 Nonlinear normal mode initialization
- 11.2 Adjoint methods
- 12. Sound Waves in Non-Hydrostatic Models
  - 12.1 Filtering sound waves with the anelastic approximation
  - 12.2 Unification of the anelastic and quasi-hydrostatic systems of equations (Arakawa and Konor 2009)
  - 12.3 Time-split integration techniques for the fully compressible non-hydrostatic equations
- 13. Thermodynamic and Dynamic Basis for Modeling the Moist Atmosphere
  - 13.1 Prognostic versus diagnostic variables in a moist model
  - 13.2 Definition of moist entropy (Ooyama 1990)
  - 13.3 Inclusion of ice
  - 13.4 Governing prognostic equations
  - 13.5 Thermodynamic diagnosis of temperature and pressure
  - 13.6 Adiabats and the many flavors of CAPE
  - 13.7 Parameterization of the precipitation process
- 14. Global Non-hydrostatic Models
  - 14.1 Global non-hydrostatic modeling at the National Centers for Environmental Prediction (NCEP, Janjic 2010)
  - 14.2 National Center for Atmospheric Research (NCAR) and the Los Alamos National Laboratory (LANL) with the Model for Prediction Across Scales (MPAS)
  - 14.3 Deutscher Wetterdienst and the Max Planck Institute for Meteorology with the ICON model
  - 14.4 Met Office with the non-hydrostatic Unified Model (UM) and the more recent New Dynamics Model (ND)
  - 14.5 Meteo-France development of the non-hydrostatic NWP model AROME
  - 14.6 Non-hydrostatic Icosahedral Atmospheric Model (NICAM, Tomita et al. 2010)
  - 14.7 Evolution of the European Center for Medium Range Weather Forecasts (ECMWF) Integrated Forecast System (IFS) to IFS/ARPEGE/ALADIN/AROME

Appendix A. Vector Formulas

Appendix B. Differential Operators in Curvilinear Coordinate Systems

Appendix C. Orthonormality of the Spherical Harmonics

Appendix D. Filters and Derivative Constraints

Appendix E. Energy and Enstrophy

Appendix F. Gauss-Legendre Quadrature

Appendix G. Exact Integration of Band-Limited Periodic Functions