

## ATS602: Atmospheric Dynamics (Spring 2011)

### Introduction

ATS602, the second semester of Atmospheric Dynamics, is based on the course notes available over the web and on the highly recommended texts listed below. The course notes (in PDF format) are available at the Schubert Research Group Web Site under "Teaching" with the user ID and password given in class. The complete list of chapters is given below. Chapters 1–10 contain material from ATS601. Selected chapters after Chapter 10 are covered in ATS602.

### Outline

1. Introduction
  - 1.1 Ideal gas law
  - 1.2 Material derivative
  - 1.3 Mass conservation
  - 1.4 Thermodynamic equation
  - 1.5 Noninertial reference frames
  - 1.6 The Foucault pendulum
  - 1.7 The complete system of equations for a dry atmosphere
2. Fundamentals
  - 2.1 Vorticity and circulation
  - 2.2 Vector vorticity equation
  - 2.3 Potential vorticity equation
  - 2.4 Impermeability theorem
  - 2.5 Circulation theorem
  - 2.6 Helicity
3. The Exact Primitive Equations
  - 3.1 Exact primitive equations in spherical coordinates
  - 3.2 Summary
4. Primitive Equations for Shallow Atmospheres
  - 4.1 Primitive equations with the traditional approximation
  - 4.2 Summary
5. The Quasi-static Primitive Equations
  - 5.1 Scale analysis
  - 5.2 Summary
6. Transformation of the Quasi-static Primitive Equations to a Generalized Vertical Coordinate
  - 6.1 The general  $\eta(p, p_s, \theta)$  coordinate
  - 6.2 Pressure coordinate
  - 6.3 Log-pressure coordinate
  - 6.4 Pseudo-height coordinate
  - 6.5 Sigma coordinate
  - 6.6 Isentropic coordinate
  - 6.7 The ECMWF hybrid vertical coordinate
  - 6.8 Arakawa-Konor hybrid  $\sigma$ - $p$  vertical coordinate
  - 6.9 Konor-Arakawa hybrid  $\theta$ - $p$ - $p_s$  vertical coordinate
7. Divergent Barotropic Primitive Equations (Shallow Water Equations)
  - 7.1 Horizontal momentum and continuity equations
  - 7.2 Potential vorticity principle for the shallow water equations
  - 7.3 Some numerical solutions
8. Nondivergent Barotropic Equations

- 8.1 From the divergent barotropic model to the nondivergent barotropic model
- 8.2 Emergence of coherent structures in two-dimensional turbulence
- 8.3 Waves and turbulence on the sphere
9. Vertical Normal Modes of a Continuously Stratified Fluid
  - 9.1 Governing equations and boundary conditions
  - 9.2 Vertical transform
  - 9.3 Solution of the Sturm-Liouville eigenproblem in the constant static stability case
  - 9.4 Summary
10. The Shallow Water Equations on an  $f$ -plane
  - 10.1 Linearization and nondimensionalization
  - 10.2 Geostrophic adjustment: One-dimensional case
  - 10.3 Case 1: An initial unbalanced wind disturbance
  - 10.4 Case 2: An initial unbalanced pressure disturbance
  - 10.5 Invertibility principle
  - 10.6 The concept of a balanced model
  - 10.7 The two-dimensional case
11. The Shallow Water Equations on an Equatorial  $\beta$ -Plane
  - 11.1 Linearization and nondimensionalization
  - 11.2 Eigenvalues and eigenfunctions
  - 11.3 An initial value problem
12. The Shallow Water Equations on the Sphere
  - 12.1 Laplace's tidal equations
  - 12.2 The nonrotating sphere
  - 12.3 Rossby-Haurwitz waves
  - 12.4 Initial value problem on the sphere
  - 12.5 More on the  $\beta$ -plane approximation
  - 12.6 Rossby waves in laboratory experiments
13. The Quasi-Geostrophic Equations
  - 13.1 Vertical coordinate and thermal wind equations
  - 13.2 Quasi-static primitive equations and quasi-geostrophic equations on an  $f$ -plane
  - 13.3 Quasi-geostrophic potential vorticity equation
  - 13.4 Kinetic energy, available potential energy, and total energy principles of quasi-geostrophic theory
  - 13.5 Two views of the omega equation
  - 13.6  $\mathbf{Q}$ -vector form of the omega equation
  - 13.7 Equivalence of the two forms of the omega equation
14. The Geostrophic Momentum Approximation and the Semi-Geostrophic Equations in Pseudo-Height Coordinates
  - 14.1 Geostrophic momentum approximation on an  $f$ -plane
  - 14.2 Geostrophic coordinates
  - 14.3 Ageostrophic circulations
  - 14.4 Comparison of semi-geostrophic and quasi-geostrophic theories
15. The Geostrophic Momentum Approximation and the Semi-Geostrophic Equations in Isentropic Coordinates
  - 15.1 Introduction
  - 15.2 Semi-geostrophic theory and the potential pseudo-density equation
  - 15.3 Invertibility principle in geostrophic space
  - 15.4 The massless layer approach
  - 15.5 Frontogenesis by horizontal deformation fields
  - 15.6 Concluding remarks
  - 15.7 Historical notes and references

16. Baroclinic Waves and Fronts
    - 16.1 Baroclinic waves and fronts in semi-geostrophic theory
    - 16.2 Baroclinic waves and fronts in the primitive equations
  17. The Ekman Layer
    - 17.1 Reynolds averaging
    - 17.2 Frictional mass transports
    - 17.3 The laminar Ekman layer (Ekman spiral)
    - 17.4 The modified Ekman layer
    - 17.5 Spin up and spin down
    - 17.6 Ekman layer in a circular vortex
  18. Barotropic Instability
    - 18.1 The Rayleigh and Fjørtoft necessary conditions for barotropic instability
    - 18.2 The simple shear layer
  19. Baroclinic Instability
    - 19.1 Quasi-geostrophic theory
    - 19.2 The Charney-Stern necessary condition for combined barotropic-baroclinic instability
    - 19.3 The Eady problem
    - 19.4 The two-layer model
  20. The Hadley Circulation and the ITCZ
    - 20.1 Introduction
    - 20.2 Derivation of the zonal mean equations
    - 20.3 The potential latitude coordinate
    - 20.4 Balanced zonal flows
    - 20.5 Invertibility principle
    - 20.6 Solutions
  21. Hurricanes
    - 21.1 Introduction
    - 21.2 Derivation of the azimuthal mean equations
    - 21.3 Balanced vortex theory and the transverse circulation equation
    - 21.4 Invertibility principle
    - 21.5 Analytic solution of the potential pseudo-density equation
    - 21.6 Numerical results
    - 21.7 Effects of PV mixing on the mean tangential flow
- Appendix A: Vector Formulas
- Appendix B: Differential Operators in Curvilinear Coordinate Systems
- Appendix C: Hermite's Equation
- Appendix D: Skew-Hermitian Property of  $\mathcal{L}$  for the Equatorial  $\beta$ -Plane
- Appendix E: Skew-Hermitian Property of  $\mathcal{L}$  on the Sphere
- Appendix F: Vector Vorticity Equation and Potential Vorticity Equation for Semi-Geostrophic Theory in Pseudo-Height Coordinates
- Appendix G: Isentropic Vorticity Equation and Potential Vorticity Equation for Semi-Geostrophic Theory in  $\theta$ -Coordinates

**Grading**

Homework	30%	
Midterm Exam	35%	(Thursday, March 10)
Final Exam	35%	(Wednesday, May 11, 9:40pm – 11:40pm)

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**Highly Recommended Texts**

- Gill, A. E., 1982: Atmosphere-Ocean Dynamics, Academic Press, 662 pages.  
 Holton, J. R., 2004: An Introduction to Dynamic Meteorology, Fourth Edition, Academic Press, 535 pages.  
 Pedlosky, J., 2003: Waves in the Ocean and Atmosphere, Springer-Verlag, 260 pages.  
 Salmon, R., 1998: Lectures on Geophysical Fluid Dynamics, Oxford University Press, 378 pages.  
 Vallis, G. K., 2006: Atmospheric and Oceanic Fluid Dynamics, Cambridge University Press, 750 pages.

An excellent introduction to dynamic meteorology can be found in Holton's textbook. The book is written at the senior undergraduate level, so all the material in it should be mastered by every atmospheric science graduate student. Gill's book is more of a graduate-level text and includes considerable oceanography. It focuses on linear theory and has an extensive discussion of fundamental fluid dynamics before the earth's rotation is introduced about a third of the way through the book (page 189). Salmon's book contains excellent discussions of many topics, such as two-dimensional turbulence, quasi-geostrophic turbulence, and semi-geostrophic theory. The newest and best text on AOFD is by Vallis. It is divided into four parts: Part I—Fundamentals; Part II—Instabilities, Wave-Mean Flow Interaction and Turbulence; Part III—Large-Scale Atmospheric Circulation; Part IV—Large-Scale Oceanic Circulation. Parts II and III are of most relevance to ATS602. Much of the material contained in Part IV is covered in Prof. Ito's oceanography course (ATS610). Although much of ATS602 is based on course notes, we shall make frequent reference to these four books. Other books which you should find useful are listed below.

**Additional Recommended Texts**

- Andrews, D. G., J. R. Holton, and C. B. Leovy, 1987: Middle Atmosphere Dynamics, Academic Press, 489 pages.  
 Baines, P. G., 1995: Topographic Effects in Stratified Flows, Cambridge University Press, 482 pages.  
 Batchelor, G. K., 1967: Fluid Dynamics, Cambridge University Press, 615 pages.  
 Charney, J. G., 1973: Planetary Fluid Dynamics, in Dynamic Meteorology, P. Morel, Editor. D Reidel, Boston, pages 99–351.  
 Cushman-Roisin, B., 1994: Introduction to Geophysical Fluid Dynamics, Prentice-Hall, 320 pages.  
 Drazin, P. G., and W. H. Reid, 1981: Hydrodynamic Stability, Cambridge University Press, 527 pages.  
 Dutton, J. A., 1976: The Ceaseless Wind, McGraw-Hill, 579 pages.  
 Eliassen, A., and E. Kleinschmidt, Jr., 1957: Dynamic Meteorology, Handbuch der Physik, Volume 48, 154 pages.  
 Green, J., 1999: Atmospheric Dynamics, Cambridge University Press, 213 pages.

- Greenspan, H. P., 1968: *The Theory of Rotating Fluids*, Cambridge University Press, 327 pages.
- Haltiner, G. J., and R. T. Williams, 1980: *Numerical Prediction and Dynamic Meteorology*, John Wiley and Sons, 477 pages.
- James, I. N., 1994: *Introduction to Circulating Atmospheres*, Cambridge University Press, 422 pages.
- Kundu, P. K., and I. M. Cohen, 2004: *Fluid Mechanics*, Third Edition. Elsevier Academic Press, 759 pages.
- Lamb, H., 1932: *Hydrodynamics*, Cambridge University Press and Dover Publications, 738 pages.
- Landau, L. D., and E. M. Lifshitz, 1959: *Fluid Mechanics*, Pergamon Press.
- LeBlond, P. H., and L. A. Mysak, 1978: *Waves in the Ocean*, Elsevier Scientific Publishing Company.
- Lighthill, M. J., 1978: *Waves in Fluids*, Cambridge University Press, 504 pages.
- Lighthill, M. J., 1986: *An Informal Introduction to Theoretical Fluid Mechanics*, Oxford University Press, 260 pages.
- Lindzen, R. S., 1990: *Dynamics in Atmospheric Physics*, Cambridge University Press, 310 pages.
- Pedlosky, J., 1987: *Geophysical Fluid Dynamics*, Second Edition, Springer-Verlag, 710 pages.
- Schey, H. M., 1997: *Div, Grad, Curl, and All That: An Informal Text on Vector Calculus*, Third Edition. Norton Publishing, 163 pages.
- Tritton, D. J., 1988: *Physical Fluid Dynamics*, Oxford Science Publications, Second Edition.
- Turner, J. S., 1973: *Buoyancy Effects in Fluids*, Cambridge University Press, 368 pages.

Although it does not contain much geophysical fluid dynamics, Batchelor's book is a classic for fundamental fluid dynamics concepts. Other excellent books on fundamental fluid mechanics are by Lamb, Landau and Lifshitz, Tritton, and Kundu and Cohen. Another classic for meteorologists is Eliassen and Kleinschmidt, especially the first three chapters (written by Eliassen). Pedlosky's book offers a rigorous and thorough treatment of many topics, including quasi-geostrophic theory and baroclinic instability. Cushman-Roisin's book is more at the level of senior undergraduates; it is enjoyable reading, with each chapter having a short biographical sketch of an important contributor to geophysical fluid dynamics. Although the Andrews et al. book primarily concerns the dynamics of the stratosphere and mesosphere, it contains excellent discussions of wave, mean-flow interaction, Eliassen-Palm fluxes, and wave activity relations, all of which are applicable to tropospheric motions. Drazin and Reid's book concentrates on hydrodynamic stability problems and is recommended (especially its section 22) for use in conjunction with Chapter 18 of the course notes. Greenspan's book has a good discussion of Ekman layers and the spin-up problem encountered in laboratory rotating fluids. Lindzen's book and Green's book are not comprehensive treatments but contain unique perspectives on selected research topics to which the authors have made fundamental contributions. For mountain wave theory and general gravity wave theory the books by Baines and by LeBlond and Mysak are useful. Turner's book concentrates on buoyancy effects in fluids; it contains good discussions of buoyant plumes and bubbles, and should be used in conjunction with Chapter 23 of the notes. Many of problems we study in dynamics are closely related to problems in numerical weather prediction, as discussed in Haltiner and Williams' book. Finally, if you need to refresh your knowledge of vector calculus (we will make frequent use gradient, divergence, and curl), the short paperback book by Schey is very good.