Atmospheric Dynamics

Fall 2008

AT601, the first semester of Atmospheric Dynamics, is based on the course notes available over the web and on the highly recommended texts listed below. The course notes (in PDF format) are available at the Schubert Research Group Web Site under "Teaching" with the user ID and password given in class. You can print out individual chapters from the website or photocopy sections using the hardcopy version of the notes that is on reserve in the ATS Branch Library. The complete list of chapters is given below. The later chapters contain material from AT602.

1. Introduction
   - 1.1 Ideal gas law
   - 1.2 Material derivative
   - 1.3 Mass conservation
   - 1.4 Thermodynamic equation
   - 1.5 Noninertial reference frames
   - 1.6 Foucault pendulum
   - 1.7 The complete system of equations for a dry atmosphere
2. Fundamentals
   - 2.1 Vorticity, vortex lines, vortex tubes
   - 2.2 Vector vorticity equation
   - 2.3 Potential vorticity equation
   - 2.4 Impermeability theorem
   - 2.5 Circulation theorem
   - 2.6 Helicity
3. Exact Primitive Equations
   - 3.1 Exact primitive equations in spherical coordinates
   - 3.2 Summary
4. Primitive Equations for Shallow Atmospheres
   - 4.1 Primitive equations with the traditional approximation
   - 4.2 Summary
5. The Quasi-static Primitive Equations
   - 5.1 Scale analysis
   - 5.2 Summary
6. Transformation of the Quasi-static Primitive Equations to a Generalized Vertical Coordinate
   - 6.1 The general $\eta(p, p_s, \theta)$ coordinate
   - 6.2 Pressure coordinate
   - 6.3 Log-pressure coordinate
   - 6.4 Pseudo-height coordinate
   - 6.5 Sigma coordinate
   - 6.6 Isentropic coordinate
   - 6.7 The ECMWF hybrid vertical coordinate
   - 6.8 Arakawa's hybrid vertical coordinate
7. Divergent Barotropic Primitive Equations (Shallow Water Equations)
   - 7.1 Horizontal momentum and continuity equations
   - 7.2 Potential vorticity principle for the shallow water equations
   - 7.3 Some numerical solutions
8. Nondivergent Barotropic Equations
   - 8.1 From the divergent barotropic model to the nondivergent barotropic model
   - 8.2 Emergence of coherent structures in two-dimensional turbulence
   - 8.3 Some numerical solutions
   - 8.4 Contour dynamics
   - 8.5 Vortex axisymmetrization and merger

8.6 Two-dimensional turbulence on the plane
8.7 Two-dimensional turbulence on the sphere: the Rhines barrier
9. Vertical Normal Modes of a Continuously Stratified Fluid
9.1 Governing equations and boundary conditions
9.2 Vertical transform
9.3 Potential vorticity principle, invertibility principle, and the final geostrophically balanced state
9.4 Heuristic development of the f-plane quasi-geostrophic equations
10. The Shallow Water Equations on an f-plane
10.1 Linearization and nondimensionalization
10.2 Geostrophic adjustment: One-dimensional case
10.3 Case 1: An initial unbalanced wind disturbance
10.4 Case 2: An initial unbalanced pressure disturbance
10.5 Invertibility principle
10.6 The concept of a balanced model
10.7 Geostrophic adjustment: Two-dimensional case
11. The Shallow Water Equations on an Equatorial β-plane
11.1 Linearization and nondimensionalization
11.2 Eigenvalues and eigenfunctions
11.3 An initial value problem
12. The Shallow Water Equations on the Sphere
12.1 Laplace's tidal equations
12.2 Rossby-Haurwitz waves
12.3 β-plane approximation
12.4 Initial value problem on the sphere
12.5 Rossby waves in laboratory experiments
13. The Quasi-Geostrophic Equations
13.1 Vertical coordinate and thermal wind equations
13.2 Quasi-static primitive equations and quasi-geostrophic equations on an f-plane
13.3 Quasi-geostrophic potential vorticity equation
13.4 Kinetic energy, available potential energy, and total energy principles of quasi-geostrophic theory
13.5 Two views of the omega equation
13.6 $\mathbf{Q}$-vector form of the omega equation
13.7 Equivalence of the two forms of the omega equation
14. The Geostrophic Momentum Approximation and the Semi-Geostrophic Equations in Pseudo-Height Coordinates
14.1 Geostrophic momentum approximation on an f-plane
14.2 Geostrophic coordinates
14.3 Ageostrophic circulations
14.4 Comparison of semi-geostrophic and quasi-geostrophic theories
14.5 Surface fronts with the uniform potential vorticity model
14.6 Upper level fronts with the discontinuous potential vorticity model
15. The Geostrophic Momentum Approximation and the Semi-Geostrophic Equations in Isentropic Coordinates
15.1 Introduction
15.2 Semi-geostrophic theory and the potential pseudo-density equation
15.3 Invertibility principle in geostrophic space
15.4 The massless layer approach
15.5 Frontogenesis by horizontal deformation fields
15.6 Concluding remarks
16. Baroclinic Waves and Fronts
17. The Ekman Layer
17.1 Reynolds averaging

17.2 Frictional mass transports
17.3 The laminar Ekman layer (Ekman spiral)
17.4 The modified Ekman layer
17.5 Spin up and spin down
17.6 Ekman layer in a circular vortex
18. Barotropic Instability
18.1 The Rayleigh and Fjørtoft necessary conditions for instability
18.2 Instability of a vorticity strip
18.3 ITCZ breakdown
19. Baroclinic Instability
19.1 Quasi-geostrophic theory
19.2 The Charney-Stern necessary condition for combined barotropic-baroclinic instability
19.3 The Eady problem
19.4 The two-layer model
20. The Hadley Circulation and the ITCZ
20.1 Derivation of the zonal mean equations
20.2 Balanced zonal flows and the meridional circulation equation
20.3 Invertibility principle
20.4 Some solutions
20.5 The primitive equation form of the Taylor relation
21. Hurricanes
21.1 Introduction
21.2 Derivation of the tangential mean equations
21.3 Balanced vortex theory and the transverse circulation equation
21.4 Invertibility principle
21.5 Effects of PV mixing on the mean tangential flow
21.6 The primitive equation form of the Taylor relation
22. Noninertial Theory of the Ocean Circulation
22.1 Spin-up of the ocean circulation
22.2 Stommel's model of the steady wind-driven circulation
22.3 Munk's model of the steady wind-driven circulation
23. Buoyant Convection
References

Appendix A. Vector Formulas
Appendix B. Differential Operators in Curvilinear Coordinate Systems
Appendix C. Hermite's Equation
Appendix D. Skew-Hermitian Property of $\mathcal{L}$ for the Equatorial β-Plane
Appendix E. Skew-Hermitian Property of $\mathcal{L}$ on the Sphere
Appendix F. Vector Vorticity Equation and Potential Vorticity Equation for Semi-Geostrophic Theory in Pseudo-Height Coordinates
Appendix G. Isentropic Vorticity Equation and Potential Vorticity Equation for Semi-Geostrophic Theory in θ-Coordinates

Grading AT601

Homework 20%
Midterm Exam 1: 25% (Friday, September 26)
Midterm Exam 2: 25% (Friday, October 31)
Final Exam 30% (Final Exam Week is December 15–19)

Highly Recommended Texts

Gill, A. E., 1982: Atmosphere-Ocean Dynamics, Academic Press, 662 pages.

Holton, J. R., 2004: An Introduction to Dynamic Meteorology, Fourth Edition, Academic Press, 535 pages.

Salmon, R., 1998: Lectures on Geophysical Fluid Dynamics, Oxford University Press, 378 pages.

Vallis, G. K., 2006: Atmospheric and Oceanic Fluid Dynamics, Cambridge University Press, 745 pages.

An excellent introduction to dynamic meteorology can be found in Holton's textbook. The book is written at the senior undergraduate level, so all the material in it should be mastered by every atmospheric science graduate student. Gill's book is more of a graduate-level text and includes considerable oceanography. It focuses on linear theory and has an extensive discussion of fundamental fluid dynamics before the earth's rotation is introduced about a third of the way through the book (page 189). Salmon's book contains excellent discussions of many topics, such as two-dimensional turbulence, quasi-geostrophic turbulence, and semi-geostrophic theory. The most recent atmospheric and oceanic fluid dynamics (AOFD) textbook is the one by G. Vallis. It has four parts: (i) fundamentals; (ii) instabilities, wave-mean flow interaction and turbulence; (iii) large-scale atmospheric circulation; (iv) large-scale oceanic circulation. Almost all the material in the first three parts is relevant to AT601/602. Although much of AT601 and AT602 is based on course notes, we shall make frequent reference to these four books. Other books which you should find useful are listed below.

Recommended Texts

Andrews, D. G., J. R. Holton, and C. B. Leovy, 1987: Middle Atmosphere Dynamics, Academic Press, 489 pages.

Baines, P. G., 1995: Topographic Effects in Stratified Flows, Cambridge University Press, 482 pages.

Batchelor, G. K., 1967: Fluid Dynamics, Cambridge University Press, 615 pages.

Charney, J. G., 1973: Planetary Fluid Dynamics, in Dynamic Meteorology, P. Morel, Editor. D Reidel, Boston, pages 99–351.

Cushman-Roisin, B., 1994: Introduction to Geophysical Fluid Dynamics, Prentice-Hall, 320 pages.

Drazin, P. G., and W. H. Reid, 1981: Hydrodynamic Stability, Cambridge University Press, 527 pages.

Dutton, J. A., 1976: The Ceaseless Wind, McGraw-Hill, 579 pages.

Eliassen, A., and E. Kleinschmidt, Jr., 1957: Dynamic Meteorology, Handbuch der Physik, Volume 48, 154 pages.

Green, J., 1999: Atmospheric Dynamics, Cambridge University Press, 213 pages.

Greenspan, H. P., 1968: The Theory of Rotating Fluids, Cambridge University Press, 327 pages.

Haltiner, G. J., and R. T. Williams, 1980: Numerical Prediction and Dynamic Meteorology, John Wiley and Sons, 477 pages.

James, I. N., 1994: Introduction to Circulating Atmospheres, Cambridge University Press, 422 pages.

Kundu, P. K., and I. M. Cohen, 2004: Fluid Mechanics, Third Edition. Elsevier Academic Press, 759 pages.

Lamb, H., 1932: Hydrodynamics, Cambridge University Press and Dover Publications, 738 pages.

Landau, L. D., and E. M. Lifshitz, 1959: Fluid Mechanics, Pergamon Press.

LeBlond, P. H., and L. A. Mysak, 1978: Waves in the Ocean, Elsevier Scientific Publishing Company.

Lighthill, M. J., 1978: Waves in Fluids, Cambridge University Press, 504 pages.

Lighthill, M. J., 1986: An Informal Introduction to Theoretical Fluid Mechanics, Oxford University Press, 260 pages.

Lindzen, R. S., 1990: Dynamics in Atmospheric Physics, Cambridge University Press, 310 pages.

Pedlosky, J., 1987: Geophysical Fluid Dynamics, Second Edition, Springer-Verlag, 710 pages.

Pedlosky, J., 2003: Waves in the Ocean and Atmosphere, Springer-Verlag, 260 pages.

Schey, H. M., 1997: Div, Grad, Curl, and All That: An Informal Text on Vector Calculus, Third Edition. Norton Publishing, 163 pages.

Tritton, D. J., 1988: Physical Fluid Dynamics, Oxford Science Publications, Second Edition.

Turner, J. S., 1973: Buoyancy Effects in Fluids, Cambridge University Press, 368 pages.

Although it does not contain much geophysical fluid dynamics, Batchelor's book is a classic for fundamental fluid dynamics concepts. Other excellent books on fundamental fluid mechanics are by Lamb, Landau and Lifshitz, Tritton, and Kundu and Cohen. Another classic for meteorologists is Eliassen and Kleinschmidt, especially the first three chapters (written by Eliassen). Pedlosky's book offers a rigorous and thorough treatment of many topics, including quasi-geostrophic theory and baroclinic instability. Cushman-Roisin's book is more at the level of senior undergraduates; it is enjoyable reading, with each chapter having a short biographical sketch of an important contributor to geophysical fluid dynamics. Although the Andrews et al. book primarily concerns the dynamics of the stratosphere and mesosphere, it contains excellent discussions of wave, mean-flow interaction, Eliassen-Palm fluxes, and wave activity relations, all of which are applicable to tropospheric motions. Drazin and Reid's book concentrates on hydrodynamic stability problems and is recommended (especially its section 22) for use in conjunction with Chapter 18 of the course notes. Greenspan's book has a good discussion of Ekman layers and the spin-up problem encountered in laboratory rotating fluids. Lindzen's book and Green's book are not comprehensive treatments but contain unique perspectives on selected research topics to which the authors have made fundamental contributions. For mountain wave theory and general gravity wave theory the books by Baines and by LeBlond and Mysak are useful. Turner's book concentrates on buoyancy effects in fluids; it contains good discussions of buoyant plumes and bubbles, and should be used in conjunction with Chapter 23 of the notes. Many of problems we study in dynamics are closely related to problems in numerical weather prediction, as discussed in Haltiner and Williams' book. Finally, if you need to refresh your knowledge of vector calculus (we will make frequent use gradient, divergence, and curl), the short paperback book by Schey is very good.