Dreams of a Stratocumulus Sleeper

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1. Foggy recollections

When we were students at UCLA in the early 1970s, the California stratus deck frequently floated over our heads. There on the beautiful campus under the clouds, we studied Lilly's (1968; hereafter L68) paper about cloud-topped mixed layers under strong inversions. The paper was recommended to us by our mentor, Prof. Akio Arakawa, who recognized the relevance of Lilly's insights to climate dynamics. Whereas spectacular supercells leap from the boundary layer to the tropopause in a single bound, L68 analyzed "wimpy" stratus and stratocumulus clouds that are only a few hundred meters thick and barely manage to precipitate. L68 was a "sleeper." It received little attention at first, but over the decades since it has picked up many citations (417 as of June 2003, not counting this Chapter), and it forms the groundwork for several currently thriving lines of research. L68's emergence as a classic research paper stems in part from the climatic importance of the cloud regimes it dealt with, but more importantly from the amazing prescience of Lilly's ideas and the clarity with which he expressed them.

Several ingredients, acquired over a number of years, came together in a two-week period during the summer of 1965 to produce the remarkable L68 paper. The first was personal experience and interest. DL's high school physics teacher ran a weather club, which DL joined with enthusiasm. After becoming the club's student leader, DL began to evolve, by his own description, into a "weather junkie, keeping daily weather records, making forecasts from local observations and testing their accuracy. When I learned to drive, I did something akin to tornado chasing, within the limits of California weather. Several times I took the car and drove up to the top of the Coast Range to the west of us, with a thermometer, and observed the big temperature increase when you go above the stratus layer." The second ingredient was a simple, accurate

conceptual model based on observational work, especially Petterssen's (1938) account of the radiative driving of California stratocumulus clouds. In his insightful paper, Petterssen argued that stratocumulus clouds are not caused by the direct cooling from the upwelling water, but rather that the boundary layer air is unstable and stratocumulus forms because of convection under the temperature inversion. Petterssen realized that outgoing radiation from the top of the moist layer is effective in maintaining the temperature inversion and the instability of the boundary layer air. The third ingredient was the mixed layer modeling approach, one of the topics of study at the second summer study program in geophysical fluid dynamics at the Woods Hole Oceanographic Institution, a program to which DL had been sent by his GFDL boss Joe Smagorinsky. This remarkable program, now approaching its 45th consecutive year, afforded DL the chance to interact with oceanographers and fluid dynamicists, in particular Stewart Turner, who was applying mixed layer modeling to the problem of the ocean thermocline. The fourth and final ingredient was moist thermodynamics, no doubt mastered by DL during his doctoral research "On the Theory of Disturbances in a Conditionally Unstable Atmosphere." A clear understanding of moist thermodynamics is crucial for generalizing dry mixed layer theory so that it can be applied to the cloud-topped boundary layer.

It is not our purpose here to comprehensively review the research edifice that the atmospheric science community has built on L68. Instead we begin, in Sections 2 and 3, by giving our perspective on why L68 is still being actively discussed 35 years after its publication. As part of this discussion, we compare modern observations of the large-scale atmospheric circulations associated with marine subtropical stratocumulus regimes with the data that was available when L68 was published. In Sections 4 and 5, we offer a new analysis of issues that were raised for the

first time by L68: the effects of radiative and evaporative cooling on entrainment. Our approach is formulated in terms of what we call the “effective inversion strength.” We conclude with some comments about the relevance of L68’s conceptual framework to the design of global circulation models.

2. Why are stratocumulus clouds so prevalent over the eastern subtropical oceans?

On the basis of data collected during research cruises, Neiburger et al. (1960) described the peculiar structure of the lower troposphere off the coast of southern California. A strong subsidence inversion hangs a few hundred meters above the sea. Below the inversion, moist-conservative variables are vertically homogenized within a “marine layer.” Especially during the summer months, a thin but remarkably persistent sheet of stratus or stratocumulus clouds hugs the inversion base. As the marine-layer air circulates towards the southwest, around the subtropical high, it encounters gradually warming sea surface temperatures (SSTs) and gradually weakening subsidence. A couple of thousand kilometers downstream from the California coast, the stratus deck breaks up into shallow cumuli.

The mean features of the Northeast Pacific stratocumulus regime, as outlined by Neiburger et al. (1960), were remarkably accurate considering the limited data available. However, an updated picture, based on QuikSCAT data, reveals many additional features. QuikSCAT streamlines and isotachs of the surface wind, averaged for July of the years 1999-2002, are shown in Fig. 1 a; the associated divergence field is shown by the color analysis in Fig. 1 b. For comparison, the July-mean divergence estimates of Neiburger et al. (their Fig. 37) are shown by the solid black isolines, also labeled in units of 10^{-6} s^{-1} . The QuikSCAT surface wind data has

Figure 1: Streamlines and isotachs of the July average surface winds (a) and the associated divergence (b). For comparison the July mean divergence estimates of Neiburger et al. are shown by the solid black isolines in (b), also labeled in units of 10^{-6} s^{-1} . These figures have been constructed from QuikSCAT data for July of the years 1999-2002 (only July 19-31 was available for 1999). In (b) the white areas adjacent to the North American coast, the Hawaiian Islands, the Aleutians, and Guadalupe are regions where the divergence could not be calculated. Courtesy of Brian McNoldy.

been resampled to 0.5 degree latitude/longitude resolution. For clarity the QuikSCAT divergence field presented in Fig. 1 b has been subjected to a 7-element boxcar average, which effectively eliminates features smaller than approximately 150 km. Fig. 1 a reveals that the strongest surface winds along the coast average more than 10 m s^{-1} , just off Cape Mendocino, California. A secondary maximum occurs just northwest of San Nicolas Island, site of the 1987 FIRE Marine Stratocumulus Experiment. Surface winds of 9 m s^{-1} are also found over a large region of the diffuent northeasterly trades east and south of Hawaii, a feature that does not appear in the isotach analysis of Neiburger et al. Another interesting feature is the shadow of light winds in the lee of the Hawaiian Islands.

Fig. 1 b reveals that the surface winds are divergent over nearly the whole area; the exceptions are the region influenced by midlatitude disturbances in the northwestern part of the figure, the region just west of the center of the anticyclone, a small region in the lee of the Hawaiian Islands, and the northern edge of the ITCZ in the southeastern part of the figure. The largest divergence is found along the North American coast, with peak values of $6.5 \times 10^{-6} \text{ s}^{-1}$. Two lobes of enhanced divergence extend westward, with a region of weak divergence between them. The northern lobe of enhanced divergence lies between 40 and 45 °N, in the surface flow accelerating eastward on the north side of the subtropical high. The southern lobe of enhanced divergence lies in the diffuent, accelerating northeasterly trades midway between Hawaii and southern California. The agreement between the QuikSCAT divergence field and the earlier estimate of Neiburger et al. is generally good, although the QuikSCAT data reveal a larger north-south extent of strong divergence along the coast and a more extensive northern lobe of strong

divergence. A QuikSCAT-based comparison of the large-scale circulations in the northeast Pacific, the southeast Pacific, the southeast Atlantic and the northeast Atlantic has recently been made by McNoldy (2003).

A few years after Neiburger's study was published, satellite imagery revealed similar cloud systems west of South America, west of Namibia in southern Africa, and to some extent west of Europe and Australia (e.g., Hubert, 1966). We began to speak of "marine subtropical stratocumulus cloud" *regimes*.

L68 explained why such regimes exist. Surface evaporation is promoted by the persistent winds of the subtropical highs. Coastal and Equatorial upwelling bring cold water to the surface, chilling the air. Boundary-layer turbulence mixes the moist cool air upward, but subsidence and the strong inversion cap the mixing. The strength and depth of the mixing are also limited by the downward near-surface sensible heat flux, which damps the turbulence. If the marine layer's top climbs high enough so that the relative humidity there reaches 100%, marine subtropical stratocumulus clouds (MSCs) form. Through mechanisms explained by L68 and discussed below, cloud formation invigorates the turbulence, and so favors a deepening of the marine layer and the cloud itself.

At the top of the marine layer, the temperature increases sharply upward by 10 K or more, and the mixing ratio of water vapor decreases upward by as much as 90%, all within an interfacial zone that is much less than 100 m thick. The inversion is a "battle-front" that marks the collision

of dry, high-potential-temperature air that has smoothly subsided from the upper troposphere, with humid, cool air that is being turbulently mixed upward from near the sea surface. “Hot-dry” is coming down, “cool-wet” is going up, and where they collide they agree to disagree.

It is amazing that the subsiding air of the subtropical high actually crosses the spectacular boundary at the marine-layer top. Turbulent *entrainment* prevents the interface itself from being advected downward by the subsidence. Entrainment is not mixing. Mixed particles move in both directions across an interface, but entrained particles move in only one direction: from the quiet air into the turbulent air. Entrainment is the active “annexation” of non-turbulent air by a growing turbulence.

As the air crosses the top of the marine layer, it is very rapidly transformed from hot-dry to cool-wet. The moistening is caused by a strong convergence of the turbulent moisture flux. The cooling is due to a strong divergence of the radiative energy flux, combined with the evaporative chilling associated with a strong convergence of the turbulent flux of liquid water; without such cooling the buoyancy force would prevent the air from descending into the marine layer.

At this point it is useful to introduce the moist static energy

$$h \equiv c_p T + gz + Lq, \quad (1)$$

where c_p is the heat capacity of air at constant pressure, T is temperature, g is the acceleration of gravity, z is height, L is the latent heat of condensation, and q is the mixing ratio of water vapor. We also define the mixing ratio of total (vapor plus liquid) water,

$$r \equiv q + l, \quad (2)$$

where l is the mixing ratio of liquid water. The definitions (1) and (2) are useful because h and r are approximately conserved under both dry adiabatic and moist adiabatic processes, and h is conserved even when precipitation is occurring.

L68 showed that the sharp turbulent and radiative flux divergences and the interfacial entrainment rate satisfy two simple relations:

$$(F_h)_B = -E\Delta\bar{h} + \Delta\bar{R}, \quad (3)$$

$$(F_r)_B = -E\Delta\bar{r}. \quad (4)$$

Here F_x is the turbulent flux of a quantity x , the subscript B denotes a level at the top of the turbulent layer, E is the entrainment mass flux, $\Delta(x)$ is the upward increase of x across the top of the marine layer, an overbar represents a horizontal average, and R is the net upward flux of energy due to radiation. A positive value of $\Delta\bar{R}$ represents intense radiative cooling within the thin interfacial layer. L68 credits Pettersen (1938) with the key insight that the longwave contribution to $\Delta\bar{R}$ (typically on the order of 50 to 100 W m⁻²) plays a key role in the physics of MSCs. L68 emphasized that by cooling the top of the marine layer, $\Delta\bar{R}$ promotes convection below. The convection effectively distributes the effects of the radiative cooling over the entire depth of the marine layer, and in so doing it also drives the turbulence of the marine layer.

The subsidence and cold water characteristic of MSC regimes are compatible with either a very shallow cloud-free marine layer or a somewhat deeper cloud-topped marine layer (Randall and Suarez, 1984). If a thin cloud forms, cloud processes including cloud-top radiative cooling promote faster entrainment that leads to a further deepening of the layer and a thicker cloud. Conversely, if a transient increase in subsidence pushes the top of the marine layer below the condensation level so that the cloud disappears, the rate of entrainment decreases, favoring a further decrease in the depth of the marine layer and tending to prevent the cloud layer from re-establishing itself.

L68 deals with interactions among turbulence, moist thermodynamics, and radiative transfer. This broad scope was necessary to encompass the processes at work in MSCs. It is remarkable that even today the coupling among these processes is completely ignored in many large-scale models.

3. How do stratocumulus clouds interact with the global circulations of the atmosphere and ocean?

With albedos that can reach 50%, MSCs scatter back to space a lot of solar radiation that would otherwise be absorbed by the oceans, thus tending to reduce the temperature of the water below. As discussed later, the cold water is favorable for the existence of the clouds. Coupled ocean-atmosphere models that fail to simulate MSCs suffer from large positive SST errors off the west coasts of the continents (e.g., Ma et al., 1994).

The cold water on the eastern side of the Pacific basin contrasts with the warm water on the western side. This SST difference induces a surface-pressure gradient that determines the strength of the low-level trade-wind flow. As discussed by Bjerknes (1966), the trades act to reinforce the SST difference. When this cooperative arrangement between the trades and the SST breaks down, an El Niño ensues, with a strong warming of the eastern-Pacific SST and a decrease in subsidence there. El Niños are associated with a decrease in subtropical marine stratocumulus clouds. This decrease in cloud cover favors a further warming of the water in the eastern Pacific, thus providing a positive feedback on the El Niño-induced warming.

As already discussed, when MSCs are present, the top of the marine layer tends to rise, so that a deeper layer of moisture is carried into the tropics. Through this mechanism, MSCs tend to increase the supply of latent heat for the deep convection of the Intertropical Convergence Zone (ITCZ).

At the same time, however, MSCs tend to produce drizzle, with precipitation rates on the order of 1 mm day^{-1} . This is about 20% of the surface evaporation rate, so drizzle from MSCs represents a non-negligible moisture sink for the trade-wind layer. The intensity of the drizzle is regulated by the availability of cloud condensation nuclei (CCN); an increase in the number density of CCN causes the available liquid water to be distributed over a larger number of smaller drops, and so decreases the drizzle rate (Albrecht, 1989). The increased number of small droplets also leads to an increase in the albedo of the clouds (Twomey et al., 1984), as can be seen very clearly in ship-tracks (e.g., Liu et al., 2000). These two “indirect effects” of aerosols on climate have excited a lot of interest, but they are not yet widely incorporated into atmospheric general circulation models (GCMs).

4. What determines the rate of entrainment at the top of a cloudy turbulent layer?

Starting with L68, theories have been proposed to determine the rate of cloud-top entrainment (e.g., Lewellen and Lewellen, 1998; Lock, 1998, 2000; Lilly, 2002 a, b); see the recent review by Stevens (2002). Many of these theories are variations on an approach suggested by L68, wherein a closure assumption is formulated in terms of the vertically integrated “buoyancy flux,” i.e., the flux of virtual temperature. The motivation for this approach is that the convection associated with the vertically integrated buoyancy flux is the primary source of the MSC’s turbulence kinetic energy (TKE). Obviously the TKE must be non-zero in order for turbulent entrainment to occur, but at the same time entrainment normally tends to reduce the buoyancy flux. All other things being equal, more TKE would imply more entrainment, but more entrainment would tend to reduce the source of TKE. This line of reasoning suggests that there is a particular entrainment rate for which the system is in balance, and L68 hypothesized that this is the entrainment rate that we observe.

Cloud-top entrainment has been a highly controversial subject, in large part because there have been very few hard facts to work with. Obviously it is important to test the entrainment theories against observations. Unfortunately, however, the entrainment rate is very difficult to measure in the field, and almost as difficult to simulate numerically, even with LES (large-eddy simulation; see the Chapter by Moeng et al. in this volume). Lacking such tests, entrainment theories have the quality of “deniable plausibility,” not to be confused with the “plausible deniability” that is so useful in the political arena. Even entrainment theories that are successfully tested against data or LES are sometimes little more than curve fits, unsupported by clear physical explanations.

Despite the difficulties, there is a consensus that L68 was correct in proposing that cloud-top radiative cooling is a powerful promoter of entrainment. There is also a (perhaps somewhat weaker) consensus that L68 was correct in proposing that cloud-top evaporative cooling promotes entrainment, although the strength of this effect is still vigorously debated.

Progress can be made by:

- finding entrainment “recipes” that work increasingly well, even if the reasons for their success are incompletely understood;
- subjecting entrainment theories to increasingly challenging observational and/or numerical tests;
- deriving entrainment theories from basic assumptions that are increasingly simple and physical.

We now report an example of the third approach, in which we attempt to clarify the mechanisms by which cloud-top radiative and evaporative cooling act to increase the entrainment rate.

Our starting point is an entrainment parameterization similar to one that has been proposed for use in the absence of clouds, following the ideas of Breidenthal and Baker (1985), Siems *et al.* (1990), and Breidenthal (1989):

$$E = \frac{b_1 \rho_B \sqrt{e_M}}{(1 + b_2 R_{i\Delta})}. \quad (5)$$

Here ρ is the density of the air, e_M is the vertically integrated TKE,

$$R_{i\Delta} = \frac{g\delta z_M \text{Max}(\Delta \overline{s_v}, 0)}{c_p \overline{T_B} e_M} \geq 0 \quad (6)$$

is a Richardson number, and b_1 and b_2 are usually assumed to be constants.

Observations and high-resolution numerical simulations of cloud-free boundary layers heated from below (e.g., Deardorff, 1974) suggest that $b_1 \cong 0.25$ and $b_2 \cong 0.25$. When there is no capping inversion, (5) simplifies to

$$E = b_1 \rho_B \sqrt{e_M}. \quad (7)$$

This special case is interesting, in part because it is so simple. It is relevant to our later discussion.

In (6), δz_M is the PBL depth, and s_v is the virtual dry static energy, which is defined by

$$s_v \equiv c_p T \left[\frac{1 + (1 + \delta)q}{1 + q + l} \right] + gz, \quad (8)$$

where $\delta \cong 0.608$ is a non-dimensional combination of the gas constants of dry air and water vapor. With this definition, s_v measures the density of the air, taking into account the possible presence of both water vapor and liquid water.

Following the spirit of L68, we write a useful approximate formula for s_v , which will be used later:

$$\begin{aligned}
s_v &\equiv c_p T \left[\frac{1 + (1 + \delta)q}{1 + q + l} \right] + gz \\
&\equiv c_p T + \delta \varepsilon L q - \varepsilon L l + gz \\
&= h - (1 - \delta \varepsilon) L q - \varepsilon L l \quad .
\end{aligned} \tag{9}$$

Here $\varepsilon \equiv \frac{c_p T}{L}$.

In (5), the “velocity scale” for the entrainment rate is $\sqrt{e_M}$. The TKE appears again in the definition of $R_{i\Delta}$. Eq. (5) is suited for use in a model that explicitly predicts e_M , as do the GCMs currently in use at Colorado State University and at the University of California at Los Angeles. The $R_{i\Delta}$ term in the denominator of (5) represents the effects of buoyancy-induced resistance to entrainment across a statically stable interface; for a given TKE, a stronger inversion leads to slower entrainment. We use the “Max” function in (6) to allow for the somewhat exotic case in which the virtual temperature actually decreases upwards across the PBL top, so that the inversion offers no resistance to entrainment. The leading term in the denominator of (5) simply protects against the case $R_{i\Delta} = 0$. The values of b_1 and b_2 can be determined by reference to standard cases for which the right answer is known. The details are omitted here, but there is evidence that $b_1 \approx 0.25$ and $b_2 \approx 0.50$.

The buoyancy flux at the top of a clear PBL satisfies

$$(F_{sv})_B = -E\Delta s_v^-. \quad (10)$$

Here the subscript B denotes values found at the base of the inversion. Substituting from (5) and (6), we obtain

$$(F_{sv})_B = \frac{-b_1 \rho_B \sqrt{e_M} \Delta s_v^-}{1 + b_2 \text{Max}\left(\frac{g \Delta s_v^- \delta z_M}{c_p \bar{T}_B e_M}, 0\right)}. \quad (11)$$

For Δs_v^- sufficiently positive, this can be approximated by

$$(F_{sv})_B \cong \frac{-b_1 \rho_B \sqrt{e_M} \Delta s_v^-}{b_2 \frac{g \Delta s_v^- \delta z_M}{c_p \bar{T}_B e_M}} = \frac{-b_1 \rho_B (e_M)^{3/2}}{b_2 \frac{g \delta z_M}{c_p \bar{T}_B}}. \quad (12)$$

According to (12), the buoyancy flux at the inversion base is negative and approximately independent of Δs_v^- , for a given value of e_M . This is true when the inversion is sufficiently strong.

From (11), we see that $(F_{sv})_B \rightarrow 0$ in the limit as $\Delta s_v^- \rightarrow 0$. For $\Delta s_v^- < 0$, (11) gives

$$(F_{sv})_B = -b_1 \rho_B \sqrt{e_M} \Delta s_v^- > 0. \quad (13)$$

This means that entrainment leads directly to convection when the entrained air is negatively buoyant.

The key hypothesis of our entrainment theory is that:

Eq. (5) holds in all cases, provided that the actual Richardson number is replaced by an “effective Richardson number” based on a suitably defined “effective inversion strength” that takes into account the effects of cloud-top cooling. (14)

Our physical interpretation of this assumption is that, for a given value of e_M , cloud-top cooling affects the entrainment rate by reducing the buoyancy of the air that is entering the tops of the downdrafts, making it easier for that air to sink.

We pursue this approach in a step-by-step fashion. First, consider a “smoke cloud,” with radiative cooling at its top, but no moisture. This useful idealization was first studied by L68, and has recently been extensively analyzed by Bretherton et al. (1999; hereafter B99). A smoke cloud feels the effects of cloud-top radiative cooling, but without the additional complications associated with the phase changes of water. Radiative cooling at the top of a smoke cloud chills the air that is being entrained across the inversion, so that the newly entrained air arrives in the upper part of the mixed layer with a cooler temperature, as if it had been entrained across a weaker inversion. This effect will be incorporated by use of an effective inversion strength in (6), the expression for the Richardson number. Blobs of radiatively chilled air drip downward into the smoke cloud under the action of the buoyancy force, thus producing an upward buoyancy flux below the cloud top, as illustrated, for example, in Fig. 8 of B99. The upward buoyancy flux favors an increase e_M . This effect is automatically taken into account in a model that predicts e_M .

To derive an expression for the effective inversion strength, we begin with an analysis of the properties of the air that is sinking in downdrafts below the inversion; the discussion is similar to one given by Randall et al. (1992). Define σ as the fractional area covered by the perturbation rising motion associated with the turbulent eddies in the PBL, and write

$$\overline{(\psi)} \equiv \sigma(\psi)_u + (1 - \sigma)(\psi)_d, \quad (15)$$

where ψ is an arbitrary intensive variable, an overbar denotes a horizontal average, and the subscripts u and d denote updrafts and downdrafts, respectively. The turbulent flux of ψ satisfies

$$F_\psi = M_c(\psi_u - \psi_d), \quad (16)$$

where M_c is a convective mass flux. The budget of $\overline{\psi}$ for the inversion layer can be expressed as

$$(F_\psi)_B = -E(\overline{\psi}_{B+} - \overline{\psi}_B) - \int_{z_B}^{z_{B+}} \overline{S_\psi} dz, \quad (17)$$

where the subscript $B+$ denotes a level just above the turbulent layer, and $\int_{z_B}^{z_{B+}} \overline{S_\psi} dz$ represents

the effects of a concentrated source or sink of $\overline{\psi}$ inside the inversion layer. Comparing (16) and (17), we find that

$$(M_c)_B[(\psi_u)_B - (\psi_d)_B] = -E(\overline{\psi}_{B+} - \overline{\psi}_B) - \int_{z_B}^{z_{B+}} \overline{S_\psi} dz. \quad (18)$$

The model used here is admittedly a crude caricature of nature, in the sense that it distinguishes only two categories of air, i.e., updrafts and downdrafts, each with distinct thermodynamic properties. Obviously a real cloud layer has a much more complex structure. Nevertheless, updrafts, downdrafts, and mass fluxes do exist in nature, and our goal is to represent them simply but explicitly in our modeling framework.

We now introduce a mixing parameter, χ_E , defined by

$$(\psi_d)_B = \chi_E \bar{\psi}_{B+} + (1 - \chi_E)(\psi_u)_B + \lambda \int_{z_B}^{z_{B+}} \overline{S_\psi} dz, \quad (19)$$

where λ is a (dimensional) coefficient to be determined below. [The definition (19) differs slightly from that used by Randall et al. (1992).] For $S_\psi \equiv 0$, (19) reduces to

$$(\psi_d)_B = \chi_E \bar{\psi}_{B+} + (1 - \chi_E)(\psi_u)_B. \quad (20)$$

An interpretation of (20) is that the air found at the top of the downdraft consists of a mixture of newly entrained above-inversion air with property $\bar{\psi}_{B+}$, and air from the top of the updraft, with property $(\psi_u)_B$. In other words, χ_E can be interpreted as the “mixing fraction” of newly entrained above-inversion air that contributes to $(\psi_d)_B$. Obviously, for $E = 0$ we expect $\chi_E = 0$. The λ term of (19) represents modification of ψ by the source/sink $\overline{S_\psi}$, before the air begins to descend at the top of the downdraft. This conceptual framework is sketched in Fig. 2.

Figure 2: Sketch used to interpret Eq. (19). See text for details.

Using (15), we can rewrite (19) as

$$(\psi_d)_B - (\psi_u)_B = \frac{\chi_E(\bar{\psi}_{B+} - \bar{\psi}_B) + \lambda \int_{z_B}^{z_{B+}} \bar{S}_\psi dz}{1 - \chi_E(1 - \sigma_B)}. \quad (21)$$

Comparison of (18) and (21) shows that

$$\left[\frac{(M_c)_B \chi_E}{1 - \chi_E(1 - \sigma_B)} - E \right] (\bar{\psi}_{B+} - \bar{\psi}_B) + \left[\frac{(M_c)_B \lambda}{1 - \chi_E(1 - \sigma_B)} - 1 \right] \int_{z_B}^{z_{B+}} \bar{S}_\psi dz = 0. \quad (22)$$

Eq. (22) holds for any ψ , including particular choices of ψ for which $S_\psi = 0$. In case $S_\psi = 0$,

(22) reduces to

$$\frac{(M_c)_B \chi_E}{1 - \chi_E(1 - \sigma_B)} - E = 0. \quad (23)$$

The quantities that appear in (23) are defined without reference to any particular ψ . Therefore, (23) must hold for any choice of ψ . We could use (23) to relate E to $(M_c)_B$ for given values of χ_E and σ_B . In particular, (23) tells us that $E = 0$ and $\chi_E = 0$ go together, as expected. For $\chi_E \ll 1$, (23) can be approximated by the beautifully simple relationship

$$(M_c)_B \chi_E \cong E. \quad (24)$$

In order that (23) be consistent with (22) in all cases, we must choose

$$\frac{(M_c)_B \lambda}{1 - \chi_E(1 - \sigma_B)} - 1 = 0, \quad (25)$$

or, comparing (23) and (25),

$$\lambda = \frac{\chi_E}{E} \cong \frac{1}{(M_c)_B}. \quad (26)$$

The second (approximate) equality in (26) applies for $\chi_E \ll 1$.

We use (15) and the first equality in (26) to rewrite (21) as

$$(\psi_d)_B - \bar{\psi}_B = \frac{\sigma_B \chi_E \left[(\bar{\psi}_{B+} - \bar{\psi}_B) + \frac{1}{E} \int_{z_B}^{z_{B+}} \bar{S}_\psi dz \right]}{1 - \chi_E (1 - \sigma_B)}. \quad (27)$$

Eq. (27) is the desired result. It is essentially an *upper boundary condition* on the properties of the air sinking in downdrafts at level B . For $\chi_E \rightarrow 0$, we get $(\psi_d)_B \rightarrow \bar{\psi}_B$ when $\bar{S}_\psi = 0$; for

$\bar{S}_\psi \neq 0$, however, $(\psi_d)_B$ can differ from $\bar{\psi}_B$ even when $\chi_E \rightarrow 0$, because $\frac{\chi_E}{E} \rightarrow \frac{1}{(M_c)_B}$ as

$\chi_E \rightarrow 0$, so that

$$(\psi_d)_B - \bar{\psi}_B \rightarrow \frac{\sigma_B}{(M_c)_B} \int_{z_B}^{z_{B+}} \bar{S}_\psi dz \text{ as } \chi_E \rightarrow 0. \quad (28)$$

Eq. (28) applies in the limiting case for which the top of the turbulent layer is an impermeable “ceiling,” with $E = 0$ and $\chi_E = 0$.

When ψ is the moist static energy, h , we can show that $\int_{z_B}^{z_{B+}} \bar{S}_h dz = -\Delta \bar{R}$, so that (27)

reduces to

$$(h_d)_B - \bar{h}_B = \frac{\sigma_B \chi_E \left(\Delta \bar{h} - \frac{\Delta \bar{R}}{E} \right)}{1 - \chi_E (1 - \sigma_B)}. \quad (29)$$

We now *define* $(\bar{h}_{B+})_{\text{eff}}$ as the “effective” value of $\bar{h}_{B+}$ in the presence of radiative cooling, i.e. it is the value that $\bar{h}_{B+}$ would have to take, *in the absence of radiative cooling*, to make $(h_d)_B$ the same as it is when radiative cooling actually is occurring. By analogy with (29), we write

$$(h_d)_B - \bar{h}_B = \frac{\sigma_B \chi_E (\Delta \bar{h})_{\text{eff}}}{1 - \chi_E (1 - \sigma_B)}, \quad (30)$$

where

$$(\Delta \bar{h})_{\text{eff}} \equiv (\bar{h}_{B+})_{\text{eff}} - \bar{h}_B. \quad (31)$$

Comparison of (29) and (30) shows that

$$(\Delta \bar{h})_{\text{eff}} = \Delta \bar{h} - \frac{\Delta \bar{R}}{E}, \quad (32)$$

so that

$$(\bar{h}_{B+})_{\text{eff}} = \bar{h}_{B+} - \frac{\Delta \bar{R}}{E}. \quad (33)$$

It follows from (32) and (9) that the effective virtual dry static energy jump for the smoke-cloud case (with $q = l = 0$) is

$$(\Delta \bar{s}_v)_{\text{eff}} = \Delta \bar{s}_v - \frac{\Delta \bar{R}}{E}. \quad (34)$$

This shows explicitly that *the effect of $\Delta\bar{R} > 0$ is to make the inversion seem weaker than it really is, in terms of the effect of entrainment on the buoyancy of the air that is sinking at the tops of the downdrafts*. The amount of radiative chilling decreases as E increases, because when entrainment is more rapid the entrained air spends less time in the layer of concentrated radiative cooling. For the smoke-cloud case,

$$(F_{sv})_B = -E\Delta\bar{s}_v + \Delta\bar{R} > 0, \quad (35)$$

and comparison with (34) shows that

$$(\Delta\bar{s}_v)_{\text{eff}} < 0. \quad (36)$$

This means that the effective inversion strength is actually negative for the smoke cloud. As shown below, however, a stronger inversion does act to reduce the rate of entrainment across the smoke-cloud top, as expected.

In accord with (14), we modify the definition of the Richardson number, Eq. (6), by using $(\Delta\bar{s}_v)_{\text{eff}}$, as defined by (34), in place of $\Delta\bar{s}_v$:

$$(R_{i\Delta})_{\text{eff}} = \frac{g\delta z_M}{c_p \bar{T}_B e_M} \left(\Delta\bar{s}_v - \frac{\Delta\bar{R}}{E} \right) < 0. \quad (37)$$

Since $(R_{i\Delta})_{\text{eff}} < 0$, (5) reduces to

$$E = b_1 \rho_B \sqrt{e_M}. \quad (38)$$

To proceed, we need to determine e_M . The steady-state TKE equation can be written as a balance between buoyant production and dissipation:

$$0 = \int_{p_{B+}}^{p_S} \frac{\kappa F_{sv}}{p} dp - C \rho_M e_M^{3/2}. \quad (39)$$

Here p is pressure, C is a nondimensional constant and ρ_M is the vertically averaged density of the air in the PBL. According to the results of large-eddy simulations reported by Moeng and Sullivan (1994), $C \approx 1$. Following B99, we consider a smoke cloud with negligible surface heat flux, i.e.,

$$(F_{sv})_S = 0. \quad (40)$$

Using (40) and (35), the buoyancy flux integral for a well mixed layer can be approximated by

$$\int_{p_{B+}}^{p_S} \frac{\kappa F_{sv}}{p} dp \approx (-E \Delta \bar{s}_v + \Delta \bar{R}) \frac{g \delta z_M}{2 c_p \bar{T}_B}. \quad (41)$$

Substituting (41) into (39), and rearranging, we find that the TKE satisfies

$$e_M^{3/2} = (\Delta \bar{R} - E \Delta \bar{s}_v) \left(\frac{g \delta z_M}{2 c_p \bar{T}_B C \rho_M} \right). \quad (42)$$

Comparing (38) and (42), we obtain a cubic equation for the entrainment rate:

$$E^3 = B(\Delta\bar{R} - E\Delta\bar{s}_v), \quad (43)$$

where, for convenience, we define

$$B \equiv \frac{(b_1 \rho_B)^3 g \delta z_M}{2 c_p \bar{T}_B C \rho_M} > 0. \quad (44)$$

For the case studied by B99, we find that $B \approx 1.83 \times 10^{-5} (\text{kg m}^{-3})^{-2}$. Eq. (43) yields a single real, positive value of E . Rewriting (46) as

$$B = \frac{E^3}{\Delta\bar{R} - E\Delta\bar{s}_v}, \quad (45)$$

makes it apparent that for sufficiently large values of B , we get

$$E \approx \Delta\bar{R} / \Delta\bar{s}_v. \quad (46)$$

The large-eddy simulations of a smoke cloud presented by B99 suggest that $E \approx 0.3 \times 10^{-3} \text{ kg m}^{-2} \text{ s}^{-1}$, for $\Delta\bar{R} \approx 60 \text{ W m}^{-2}$ and $\Delta\bar{s}_v \approx 7000 \text{ J kg}^{-1}$. To make (43) agree with the LES results, we have to choose $b_1 \approx 0.0034$, which is much smaller than the value (0.25) inferred from simulations of the cloud-free boundary layer heated from below. We hypothesize that the implied variation of b_1 is real, and is due to physical differences between the idealized smoke-cloud case,

in which the turbulence is driven entirely by cooling from above, and a clear boundary layer that is driven entirely by heating from below. This hypothesis can be explored through additional LES-based research.

We now extend our entrainment closure to the more geophysically relevant case of a water cloud, for which both radiative and evaporative cooling occur at cloud top. Following the approach of L68, we use the fact that in a uniformly saturated layer isobaric fluctuations of water vapor and moist static energy are approximately proportional to each other, i.e.

$$Lq' \cong \left(\frac{\gamma}{1 + \gamma} \right) h', \quad (47)$$

where

$$\gamma \equiv \frac{L}{c_p} \left(\frac{\partial q}{\partial T} \right)_p^*. \quad (48)$$

By combining (47) with (9), we can show that

$$(s_v)_{d,B} - (\bar{s}_v)_B = \beta(h_{d,B} - \bar{h}_B) - \varepsilon L(r_{d,B} - \bar{r}_B), \quad (49)$$

where

$$\beta \equiv \frac{1 + (1 + \delta)\gamma\varepsilon}{1 + \gamma} \quad (50)$$

is a positive nondimensional coefficient. It is important to note that (49) applies only when the layer is uniformly cloudy; if there are holes in the cloud, (49) must be replaced by a more complicated equation, as discussed in Section 5 below.

By analogy with (29), the total water mixing ratio, r , in the downdraft satisfies

$$(r_d)_B - \bar{r}_B = \left[\frac{\sigma_B \chi_E}{1 - \chi_E(1 - \sigma_B)} \right] \Delta \bar{r}. \quad (51)$$

Substitute into (49) from (29) and (51) to obtain

$$\begin{aligned} (s_v)_{d,B} - (\bar{s}_v)_B &= \beta \left[\frac{\sigma_B \chi_E \left(\Delta \bar{h} - \frac{\Delta \bar{R}}{E} \right)}{1 - \chi_E(1 - \sigma_B)} \right] - \varepsilon L \left[\frac{\sigma_B \chi_E}{1 - \chi_E(1 - \sigma_B)} \right] \Delta \bar{r} \\ &\equiv \left[\frac{\sigma_B \chi_E}{1 - \chi_E(1 - \sigma_B)} \right] \left[[\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}] - \beta \frac{\Delta \bar{R}}{E} \right], \end{aligned} \quad (52)$$

where

$$\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}} \equiv \beta \Delta \bar{h} - \varepsilon L \Delta \bar{r} \quad (53)$$

is a notation introduced by Randall (1980), who showed that

$$(\Delta \bar{s}_v)_{\text{crit}} = \left[\frac{1 - (1 + \delta)\varepsilon}{1 + \gamma} \right] L(q_{B+}^* - q_{B+}) \quad (54)$$

is a positive measure of the dryness of the air above the inversion. In (54), q^* denotes the saturation mixing ratio, and the expression in [] is nondimensional and positive. In a stratocumulus regime, $(\Delta \bar{s}_v)_{\text{crit}}/c_p$ is typically on the order of 5 K.

To identify the “effective” inversion strength and effective Richardson number, we *define* $(\Delta \bar{s}_v)_{\text{eff}}$ by

$$(s_v)_{d,B} - (\bar{s}_v)_B = \left[\frac{\sigma_B \chi_E}{1 - \chi_E(1 - \sigma_B)} \right] (\Delta \bar{s}_v)_{\text{eff}}, \quad (55)$$

where $(\Delta \bar{s}_v)_{\text{eff}} \equiv (\bar{s}_{vB+})_{\text{eff}} - \bar{s}_{vB}$. Comparison of (52) and (55) gives

$$(\Delta \bar{s}_v)_{\text{eff}} = [\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}] - \beta \frac{\Delta \bar{R}}{E}. \quad (56)$$

Eq. (56) shows that the effective inversion strength is reduced by evaporative cooling, by an amount $(\Delta \bar{s}_v)_{\text{crit}}$, and that it is further reduced by radiative cooling.

Replacing $\Delta \bar{s}_v$ by $(\Delta \bar{s}_v)_{\text{eff}}$ in (6), and substituting the result into (5), we obtain

$$E = \frac{b_1 \rho_B \sqrt{e_M}}{1 + b_2 \frac{g \delta z_M}{c_p \bar{T}_B e_M} \text{Max} \left\{ [\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}] - \frac{\beta \Delta \bar{R}}{E}, 0 \right\}}, \quad (57)$$

which can be rearranged to

$$E = \begin{cases} \frac{b_1 \rho_B \sqrt{e_M} + b_2 \left(\frac{g \delta z_M}{e_M} \right) \frac{\beta \Delta \bar{R}}{c_p \bar{T}_B}}{1 + b_2 \left(\frac{g \delta z_M}{e_M} \right) \frac{[\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}]}{c_p \bar{T}_B}} & \text{for } [\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}] - \frac{\beta \Delta \bar{R}}{E} \geq 0 \\ b_1 \rho_B \sqrt{e_M} & \text{for } [\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}] - \frac{\beta \Delta \bar{R}}{E} < 0 \end{cases} . \quad (58)$$

We can show that

$$[\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}] - \frac{\beta \Delta \bar{R}}{E} \geq 0 \quad \text{is equivalent to} \quad [\Delta \bar{s}_v - (\Delta \bar{s}_v)_{\text{crit}}] b_1 \rho_B \sqrt{e_M} \geq \beta \Delta \bar{R}. \quad (59)$$

The effect of evaporative cooling is to reduce the denominator of (58). The effect of radiative cooling is to increase the numerator, just as in the case of the smoke cloud.

5. What are the processes that cause MSCs to break up into shallow cumuli on their western and equatorward boundaries?

If the entrainment problem has been controversial, the stratocumulus-break-up problem has been even more so. Many mechanisms can lead to the destruction of a stratocumulus cloud layer, including increased subsidence and horizontal advection. One possible mechanism for the destruction of a uniform cloud layer is “cloud-top entrainment instability” (CTEI). The concept was discussed by L68, who suggested that if the inversion is not sufficiently strong, evaporatively enhanced entrainment can “run away,” leading to the evaporative disruption of the uniform cloud layer. The concept of CTEI has elicited a lot of interest, and at least as much skepticism.

Randall (1980) showed that the cloud-top buoyancy flux in a uniformly cloudy layer satisfies

$$(F_{sv})_B = -E[\Delta\bar{s}_v - (\Delta\bar{s}_v)_{\text{crit}}] + \beta\Delta R. \quad (60)$$

As can be seen from (60), when

$$\Delta\bar{s}_v < (\Delta\bar{s}_v)_{\text{crit}}, \quad (61)$$

entrainment will promote a positive buoyancy flux at the inversion base. Randall (1976, 1980) and Deardorff (1980) argued that such a positive buoyancy flux will generate additional TKE, thus promoting further entrainment, and leading to a runaway destruction of the cloud through rapid infusion of dry air, i.e., to CTEI. They hypothesized that (61) is the criterion for the onset of CTEI.

Observations (e.g., Kuo and Schubert, 1988) do not support this hypothesis; they indicate that the entrainment rate remains modest when (61) is satisfied, and that the cloud layer can survive intact or nearly so. Large-eddy simulations (LES) by Moeng (2000) agree that the entrainment rate remains small when (61) is satisfied, but in contrast to the observations Moeng's results show that the fractional cloudiness decreases as $[\Delta\bar{s}_v - (\Delta\bar{s}_v)_{\text{crit}}]$ becomes increasingly negative. We have no explanation for the apparent discrepancy between the observations and the LES results.

Before completing our discussing of entrainment parameterization, we present some stratocumulus simulations performed with the Colorado State University General Circulation Model. The model is described by Ringler et al. (2000). In the simulation used here, the model is running in pure climate mode; no observed initial conditions were used. The entrainment rate has been parameterized by the method described here. When inequality (61) is satisfied, CTEI is assumed to occur and the stratocumulus cloudiness is set to zero. Fig. 3 shows the model results and, for comparison, observations from Warren et al. (1986, 1988). The model successfully simulates the primary marine stratocumulus regimes found west of North America, South America, and southern Africa, as well as secondary marine stratocumulus regimes in such places as the North Atlantic. Experiments have shown that if the CTEI criterion is ignored, the model produces much more extensive and less realistic boundary-layer stratus clouds.

Moeng's (2000) simulations of CTEI raise an important but neglected point: Any theory of CTEI should allow for partial cloudiness. It should therefore come as no surprise that Eq. (60), which is based on the assumption of uniform cloudiness, cannot describe the break-up of a uniform cloud layer. In the context of the model presented in Section 4 of this Chapter, a broken cloud layer has to be represented in terms of cloudy updrafts and clear downdrafts. We now derive an expression for the rate of entrainment across the top of such a partly cloudy layer. Our starting point is

$$(s_v)_{d,B} - (\bar{s}_v)_B = (h_{d,B} - \bar{h}_B) - (1 - \delta\epsilon)L(r_{d,B} - \bar{r}_B) + [1 - (1 + \delta)\epsilon]L(l_{d,B} - \bar{l}_B), \quad (62)$$

which can readily be obtained from (9). For the case of a partly cloudy layer, with a cloudy updraft and a cloud-free downdraft,

Figure 3: The top panel shows the July PBL stratus cloud amount as simulated with the Colorado State University general circulation model. The lower panel shows corresponding observations from Warren et al. (1986, 1988). The observations are not available for Antarctica and the adjacent southern ocean.

$$l_{d,B} = 0 \text{ and } \dot{l}_B = \sigma_B l_{u,B}, \quad (63)$$

so that (62) can be simplified to

$$(s_v)_{d,B} - (\bar{s}_v)_B = (h_{d,B} - \bar{h}_B) - (1 - \delta\varepsilon)L(r_{d,B} - \bar{r}_B) - [1 - (1 + \delta)\varepsilon]L\sigma_B l_{u,B}. \quad (64)$$

Substituting from (29) and (51), we get

$$\begin{aligned} & (s_v)_{d,B} - (\bar{s}_v)_B \\ &= \frac{\sigma_B \chi_E \left(\Delta \bar{h} - \frac{\Delta \bar{R}}{E} \right)}{1 - \chi_E(1 - \sigma_B)} - (1 - \delta\varepsilon)L \left[\frac{\sigma_B \chi_E}{1 - \chi_E(1 - \sigma_B)} \right] \Delta \bar{r} - [1 - (1 + \delta)\varepsilon]L\sigma_B l_{u,B} \end{aligned} \quad (65)$$

Comparing (65) with (55), we find that

$$\begin{aligned} & \frac{\sigma_B \chi_E \left(\Delta \bar{h} - \frac{\Delta \bar{R}}{E} \right)}{1 - \chi_E(1 - \sigma_B)} - (1 - \delta\varepsilon)L \left[\frac{\sigma_B \chi_E}{1 - \chi_E(1 - \sigma_B)} \right] \Delta \bar{r} - [1 - (1 + \delta)\varepsilon]L\sigma_B l_{u,B} \\ &= \frac{\sigma_B \chi_E (\Delta \bar{s}_v)_{\text{eff}}}{1 - \chi_E(1 - \sigma_B)}, \end{aligned} \quad (66)$$

which is equivalent to

$$(\Delta \bar{s}_v)_{\text{eff}} = \Delta \bar{s}_v - \left(\frac{1 - \chi_E}{\chi_E} \right) [1 - (1 + \delta)\varepsilon]L l_{u,B} - \frac{\Delta \bar{R}}{E}. \quad (67)$$

Eq. (67) is analogous to (56).

For $\chi_E \ll 1$, we can use (24) to approximate (67) by

$$(\Delta \bar{s}_v)_{\text{eff}} \cong \Delta \bar{s}_v - \frac{1}{E} \left\{ (M_c)_B [1 - (1 + \delta)\varepsilon] L l_{u,B} + \Delta \bar{R} \right\}. \quad (68)$$

Since $(M_c)_B l_{u,B}$ is the liquid water flux at level B , Eq. (68) states that the effective inversion strength is reduced by the cooling due evaporation of the liquid water carried upward to the PBL top, as well as by the radiative cooling:

$$(\Delta \bar{s}_v)_{\text{eff}} \cong \Delta \bar{s}_v - \frac{1}{E} \left\{ [1 - (1 + \delta)\varepsilon] L (F_l)_B + \Delta \bar{R} \right\}. \quad (69)$$

Using the approximation (68), we replace $\Delta \bar{s}_v$ by $(\Delta \bar{s}_v)_{\text{eff}}$ in (6), and substitute the result into (5), to obtain

$$E \cong \frac{b_1 \rho_B \sqrt{e_M}}{1 + b_2 \frac{g \delta z_M}{c_p \bar{T}_B e_M} \text{Max} \left(\Delta \bar{s}_v - \frac{1}{E} \left\{ [1 - (1 + \delta)\varepsilon] L (F_l)_B + \Delta \bar{R} \right\}, 0 \right)}, \quad (70)$$

which can be rearranged to

$$E \equiv \begin{cases} \frac{b_1 \rho_B \sqrt{e_M} + b_2 \frac{g \delta z_M}{c_p \bar{T}_B e_M} \left\{ [1 - (1 + \delta) \varepsilon] L(F_l)_B + \Delta \bar{R} \right\}}{1 + b_2 \frac{g \delta z_M}{c_p \bar{T}_B e_M} \Delta \bar{s}_v} & \text{for } (\Delta \bar{s}_v)_{\text{eff}} \geq 0 \quad , \\ b_1 \rho_B \sqrt{e_M} & \text{for } (\Delta \bar{s}_v)_{\text{eff}} < 0 \quad . \end{cases} \quad (71)$$

By substituting (71) into (68), we can show that

$$(\Delta \bar{s}_v)_{\text{eff}} \geq 0 \text{ is equivalent to } \Delta \bar{s}_v \geq \frac{[1 - (1 + \delta) \varepsilon] L(F_l)_B + \Delta \bar{R}}{b_1 \rho_B \sqrt{e_M}}. \quad (72)$$

To our knowledge, (71) is the first equation proposed to determine the rate of entrainment at the top of a partly cloudy layer. It is suitable for use in a model that is capable of determining $(F_l)_B$ in a partly cloudy layer; an example is ADHOC, which is described by Lappen and Randall (2001). We envision a modified version of ADHOC in which the PBL top and the entrainment rate are introduced as explicit parameters.

A question can be raised about the liquid water flux, $(F_l)_B$, in a partly cloudy layer. Should it not depend on the entrainment rate, as $(F_h)_B$ and $(F_r)_B$ do, and as $(F_l)_B$ does in a fully cloudy layer? If $(F_l)_B$ did depend on E , then the entrainment rate would enter implicitly on the right-hand side of (71), which would therefore have to be modified in order to give a solution for E . We claim that in a partly cloudy layer $(F_l)_B$ is independent of E , so that (71) can be used as given. Our reasoning is that although the entrained air enters and alters the properties of the clear

downdrafts, it has no direct effect on the cloudy updrafts. The liquid water mixing ratio in the downdrafts is zero by definition in a partly cloudy layer, and this is true regardless of the entrainment rate. The liquid water mixing ratio and other properties of the air at the tops of the updrafts are determined by processes at lower levels in the updrafts, and are not directly affected by entrainment. We can therefore determine $(F_l)_B$ without knowing the entrainment rate.

6. **Conclusion: The importance of cloud-scale process coupling for large-scale cloudiness**

Thanks in large part to L68, we have a reasonably good understanding of why MSCs exist. It is therefore somewhat surprising that GCMs have had limited success in simulating them. One reason for this unfortunate situation is that MSCs are produced by closely coupled turbulent, radiative and microphysical processes. GCMs, for the most part, fail to represent this coupling. In fact, there is a trend today towards “modularization” of GCMs. In a modularized GCM, the various physical parameterizations are segregated into software “compartments” (subroutines, etc.), like animals in a zoo. The parameterizations communicate only through their mutual effects on the shared “large-scale environment,” which is predicted by the GCM.

From a software engineering point of view, modularization has a certain appeal, but model development is not software engineering. One of the most revolutionary insights of L68 is that the interactions among turbulence, radiation, and phase changes are tightly coupled on small scales. For example, the turbulent entrainment rate is strongly affected by both cloud-top radiative cooling and the effects of phase changes on the buoyancy flux. The turbulence extends as high as it does only because of this coupling between the cloud layer and the turbulence. These couplings occur on small space and time scales, and *not* merely through their mutual interactions with the

large-scale environment. Failure to account for such cloud-scale couplings has been a major obstacle to the simulation of MSCs with GCMs. There are many other examples of process-coupling that argue for less, not more modularity in our models (e.g., Arakawa, 2003). In its sophisticated coupling of radiative, turbulent, and microphysical processes, L68 was (at least) 35 years ahead of its time.

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References

- Albrecht, B. A., 1989: Aerosols, cloud microphysics, and fractional cloudiness. *Science*, **245**, 1227-1230.
- Arakawa, A., 2003: The cumulus parameterization problem: Past, present, and future. Submitted to *J. Climate*.
- Bjerknes, J., 1966: A possible response of the atmospheric Hadley circulation to equatorial anomalies of ocean temperature. *Tellus*, **18**, 820-829.
- Bretherton, C. S., M. K. MacVean, and 14 coauthors, 1999: An intercomparison of radiatively driven entrainment and turbulence in a smoke cloud, as simulated by different numerical models. *Quart. J. Roy. Meteor. Soc.*, **125**, 391-423.
- Deardorff, J. W., 1974: Three-dimensional numerical study of the height and mean structure of a heated planetary boundary layer. *Bound. Layer Meteor.*, **7**, 81-106.
- Deardorff, J. W., 1980: Cloud top entrainment instability. *J. Atmos. Sci.*, **37**, 131-147.
- Hubert, L. F., 1966: Mesoscale cellular convection. *Meteorological Satellite Laboratory Report No. 37*, Washington, DC, 68 pp.
- Kuo, H.-C., and W. H. Schubert, 1988: Stability of cloud-topped boundary layers. *Quart. J. Roy. Meteor. Soc.*, **114**, 887-916.

- Lappen, C.-L., and D. A. Randall, 2001: Towards a unified parameterization of the boundary layer and moist convection. Part I. A new type of mass-flux model. *J. Atmos. Sci.*, **58**, 2021-2036.
- Lewellen, D., and W. Lewellen, 1998: Large-eddy boundary layer entrainment. *J. Atmos. Sci.*, **55**, 2645-2665.
- Lilly, D. K., 1968: Models of cloud-topped mixed layers under a strong inversion. *Quart. J. Roy. Meteor. Soc.*, **94**, 292-309.
- Lilly, D. K. 2002 a: Entrainment into mixed layers. Part I: Sharp-edged and smoothed tops. *J. Atmos. Sci.*, **59**, 3340–3352.
- Lilly, D. K. 2002 b: Entrainment into mixed layers. Part II: A new closure. *J. Atmos. Sci.*, **59**, 3353–3361.
- Liu, Q., Y. L. Kogan, D. K. Lilly, D. W. Johnson, G. E. Innis, P. A. Durkee, and K. E. Nielsen, 2000: Modeling of ship effluent transport and its sensitivity to boundary layer structure. *J. Atmos. Sci.*, **57**, 2779–2791.
- Lock, A. P., 1998: The parameterization of entrainment in cloudy boundary layers. *Quart. J. Roy. Meteor. Soc.*, **124**, 2729-2753.
- Lock, A. P., and M. K. MacVean, 1999: The parameterization of entrainment driven by surface heating and cloud-top cooling. *Quart. J. Roy. Meteor. Soc.*, **125**, 271-299.

- Lock, A. P., 2000: The numerical representation of entrainment in parameterizations of boundary layer turbulent mixing. *Mon Wea. Rev.*, **129**, 1148-1163.
- Ma, C.-C., C. R. Mechoso, A. Arakawa, and J. D. Farrara, 1994: Sensitivity of a coupled ocean-atmosphere model to physical parameterizations. *J. Climate*, **7**, 1883-1896.
- Moeng, C.-H., and P. P. Sullivan, 1994: A comparison of shear- and buoyancy-driven planetary boundary layer flows. *J. Atmos. Sci.*, **51**, 999-1022
- Moeng, C.-H., 2000: Entrainment rate, cloud fraction, and liquid water path of PBL stratocumulus clouds. *J. Atmos. Sci.*, **57**, 3627-3643.
- Neiburger, M., D. S. Johnson, and C.-W. Chien, 1961: *Studies of the structure of the atmosphere over the eastern Pacific ocean in summer. I. The inversion over the eastern north Pacific ocean*. University of California Press. Berkeley and Los Angeles.
- Petterssen, S., 1938: On the causes and the forecasting of the California fog. *Bull. Amer. Meteor. Soc.*, **19**, 49--55.
- Randall, D. A., 1976: *The interaction of the planetary boundary layer with large-scale circulations*. Ph.D. thesis, UCLA, 276 pp.
- Randall, D. A., 1980: Conditional instability of the first kind, upside-down. *J. Atmos. Sci.*, **37**, 125-130.

- Randall, D. A., and M. J. Suarez, 1984: On the dynamics of stratocumulus formation and dissipation. *J. Atmos. Sci.*, **41**, 3052-3057.
- Randall, D. A., 1987: On the turbulent fluxes of liquid water and buoyancy in partly cloudy layers. *J. Atmos. Sci.*, **44**, 850-858.
- Randall, D. A., Q. Shao, and C.-H. Moeng 1992: A Second-Order Bulk Boundary-Layer Model. *J. Atmos. Sci.*, **49**, 1903-1923.
- Ringler, T. D., R. P. Heikes, and D. A. Randall, 2000: Modeling the atmospheric general circulation using a spherical geodesic grid: A new class of dynamical cores. *Mon. Wea. Rev.*, **128**, 2471-2490.
- Stevens, B., 2002: Entrainment in stratocumulus-topped mixed layers. *Quart. J. Roy. Meteor. Soc.*, **128**, 2663-2690.
- Stevens, B., D. H. Lenschow, and co-authors, 2003: Dynamics and chemistry of marine stratocumulus -- DYCOMS-II. *Bull Amer. Meteor. Soc.*, **84**, 579-593.
- Twomey, S. A., M. Piepgrass, and T. L. Wolfe, 1984: An assessment of the impact of pollution on global cloud albedo. *Tellus*, **36**, 356-366.
- Warren, S. G., C. J. Hahn, J. London, R. M. Chervin, and R. L. Jenne, 1986: Global distribution of total cloud cover and cloud type amounts over land. *NCAR/TN-273+STR*, 29 pp. and 199 maps.

Warren, S. G., C. J. Hahn, J. London, R. M. Chervin, and R. L. Jenne, 1988: Global distribution of total cloud cover and cloud type amounts over the ocean. *NCAR/TN-317+STR*, 40 pp. and 170 maps.