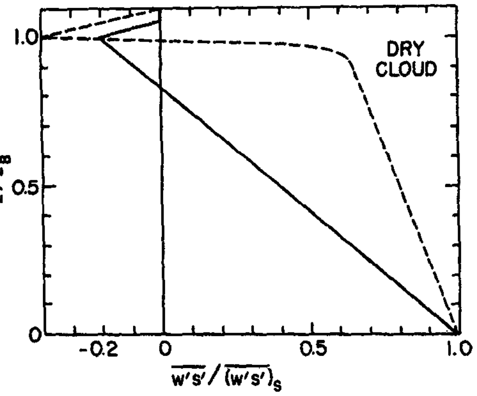

FIG. 1. Vertical profiles of $\overline{w's'}$, normalized by its surface value $(\overline{w's'})_S$, for the dry cloud. The solid curve represents the buoyancy flux profile with zero or constant radiative cooling. The dashed curve includes strong radiative cooling near the top of the bounday layer. The e-folding depth (optical depth) for emission has been taken equal to 0.016 z_B and $F_B = (\overline{w's'})_S$. In order to better display the curves near $z = z_B$, the uppermost height at which $\overline{w's'} = 0$ has been shifted upward slightly.

unphysical. They then prescribe a positive buoyancy flux at z_B which is about five times the magnitude of the negative surface buoyancy flux [see their Eq. (2.7b)]. If strong divergence of net irradiance exists near $z = z_B$ this prescription may coincidentally be approximately correct just below $z = z_B$. However, the surface buoyancy flux should never be used to diagnose the radiative divergence at the stratocumulus top.

It is our contention, in view of the above arguments, that the effects of radiation on the cloud capped boundary layer have been accounted for inadequately.

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REFERENCES

Deardorff, J. W., 1976: On the entrainment rate of a stratocumulus-topped mixed layer. *Quart. J. Roy. Meteor. Soc.*, **102**, 563–582.

Kahn, P. H., and J. A. Businger, 1979: The effect of radiative flux divergence on entrainment of a saturated convective boundary layer. *Quart. J. Roy. Meteor. Soc.*, **105**, 303–305.

Schubert, W. H., J. S. Wakefield, E. J. Steiner and S. K. Cox, 1979: Marine stratocumulus convection, Part I: Governing equations and horizontally homogeneous solutions. *J. Atmos. Sci.*, **36**, 1286–1307.

The Effects of Radiative Cooling in a Cloud-Topped Mixed Layer

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ABSTRACT

The sensitivity of models of cloud-topped mixed layers to various specifications of the radiative cooling rate near the cloud top is investigated. It is found that for the "dry cloud" case an assumed distributed cooling rate leads to a shallower mixed layer and more positive surface turbulent heat flux than does a discontinuous radiative flux model.

The purpose of this note is to try to clarify the effects of infrared radiation on the turbulent fluxes and entrainment into a cloud-topped mixed layer. An earlier model for such layers (Lilly, 1968) involved the assumption that the radiatively active layer was very thin compared to the depth of the total mixed layer and that the effects of radiative cooling could then be incorporated into the cloud-top jump condition. Deardorff (1976), Kahn and Businger (1979) and Deardorff and Businger (1980) argue that this assumption is at least conceptually deficient, in that it does not allow radiative cooling to generate turbulent mixing properly. They also suggest that the assumption may lead to inaccurate predictions of the entrainment rate and other properties of the layer.

As an improvement on Lilly's assumption, Kahn and Businger propose to replace the radiative flux

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discontinuity at cloud top by a finite cooling rate distributed within the upper part of the cloud layer according to the opacity of the cloud. Deardorff develops a somewhat more complex model, with an upper transition layer simulating the observed irregularity of cloud tops and a radiative flux divergence layer extending further down. We interpret the difference between these approaches as that between use of either strictly Eulerian coordinates (Deardorff) or a natural vertical coordinate whose origin is defined by the local cloud top. Schubert *et al.* (1979) apply both the original Lilly model and a partially distributed radiative cooling formulation, consisting of a discontinuity of part of the flux at the cloud top with a constant flux divergence in the mixed layer. Randall (1980) uses a similar formulation differing in that the radiative cooling is confined to a fixed depth of cloud, not to the entire mixed layer.

We here principally address the quantitative aspect of the Deardorff-Kahn-Businger arguments, i.e., what are the practical differences between predictions of the Lilly, Schubert and Kahn-Businger models. From reconsideration of the abovementioned papers and correspondence with some of the authors in connection with this note, it appears, however, that the conceptual problem is of at least equal concern. Therefore, we attempt to address that aspect, also, though perhaps with less definitive results, as it seems to involve personal philosophy and taste.

The conceptual problem, originally noted to a certain extent by Lilly, involves an incomplete association between positive buoyancy flux, which is necessary to generate turbulence, and entrainment at the cloud top. When the radiative flux divergence region is modeled as a flux discontinuity, the region of negative turbulent heat flux which must balance it shrinks to a spike and no compensating zone of positive heat flux appears at lower levels. Thus, entrainment of warm air into the cloud layer from above its top occurs without an apparent source of kinetic energy. While this is physically inconsistent, the magnitude of the error depends on the scale of the entrainment process. If entrainment plumes are very small and short-lived, only a slight amount of energy is required to draw warm air down into the mixed layer. The entrained air can then be cooled by radiation. Thus it is suggested that when surface heat flux is small the scale of the entrainment process is determined by the radiation scale. In his recent analysis of three-dimensional numerical simulations of cloud-topped mixed layers Deardorff (1979) obtained results consistent with this hypothesis, although he did not attempt to explore the full range of the parameters which define the problem. We therefore believe that the original Lilly model remains as a valid limit for the case of high cloud opacity, and that it is appropriate to test the validity of that limit quantitatively without excessive concern for its apparent conceptual defects.

In this paper we show results of calculations of the evolution and steady-state structure of cloud-filled boundary layers using the Lilly discontinuous flux assumption (L), the Kahn-Businger (KB) assumption and that by Schubert *et al.* (S). Our calculations are confined to the "dry cloud" model, as introduced by Lilly, which may be regarded as representing a radiatively dense smoke cloud filling the mixed layer. The effects of condensation and evaporation are important in real cloud layers, however, so that our results are incomplete in this respect.

Lilly's basic assumption for the "dry cloud" mixed layer was, and remains for this work, the constancy of mean potential temperature $\bar{\theta}$ within it. Thus for a horizontally homogeneous layer the mean θ equation may be written

$$\partial\bar{\theta}/\partial t = -(\overline{w'\theta'} + F)\big|_0^z z^{-1}, \tag{1}$$

where $\overline{w'\theta'}$ and F are, respectively, the upward turbulent and net radiative fluxes of θ. The height z is assumed to be less than that of the top of the mixed layer, denoted by H.

The turbulent fluxes are defined by two critical, and perhaps always somewhat ambiguous, assumptions. At the surface a transfer coefficient expression is assumed, i.e.,

$$(\overline{w'\theta'})_0 = C_T V(\theta_s - \bar{\theta}), \tag{2}$$

where θ_s is the surface potential temperature, V the mixed-layer wind speed, assumed constant with height, and C_T the heat transfer coefficient, assumed constant or at most a function of V and $\theta_s - \bar{\theta}$. The turbulent flux above the surface is confined by a turbulent energy balance assumption, which we write in the form introduced by Kraus and Schaller (1978), i.e.,

$$\int_{\text{negative}} \overline{w'\theta_v'}dz = -k^2 \int_{\text{positive}} \overline{w'\theta_v'}dz, \tag{3}$$

where the left and right integrals are the total negative and positive contributions to the buoyancy flux integral, respectively. We will here assume that fluctuations of θ_v, the virtual potential temperature, are identical with those of θ. The implication of Eq. (3) is that a fraction $1 - k^2$ of the turbulent energy generated by positive buoyancy flux is dissipated, while the remainder is transported into regions of negative buoyancy flux and there transformed back to potential form. Thus k might be termed a coefficient of energy transport efficiency. In Lilly's original formulation only the extreme cases, $k = 0$ and 1, were treated in detail. Since then k has been estimated, from the results of laboratory

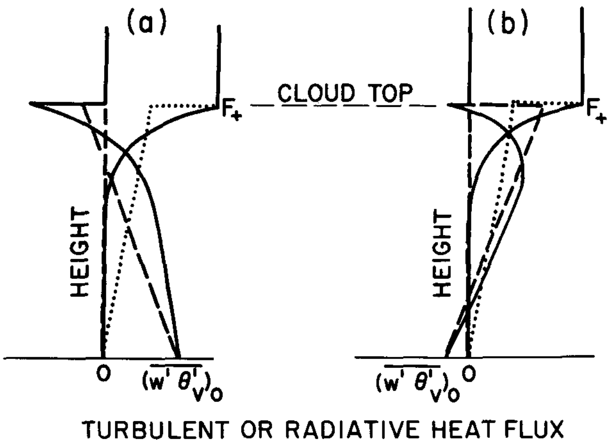

FIG. 1. Turbulent and radiative heat flux profiles for the Kahn-Businger model (solid), the Lilly discontinuous flux model (dashed) and the Schubert model (dotted) for a "dry cloud" layer with the radiative cooling. Curves (a) are for positive and (b) for negative surface heat flux. The radiative flux curves all start at the origin and attain a value of F_+ at the cloud top and above, while the turbulent flux curves start at $(\overline{w'\theta'_v})_0$ and jump discontinuously to zero at the cloud top.

experiments, numerical simulation and field data analysis, to lie in the range 0.2–0.3. There is no strong reason to expect it to be a universal constant, however. Deardorff (1979) shows evidence that k might vary 50% when condensation heating and radiative cooling are important factors.

Perhaps the principal problems with Eqs. (2) and (3), and for that matter (1), are likely to occur in conditions of rapidly changing surface temperature, especially when the surface becomes colder than the mixed layer. In such a situation the surface layer may become almost nonturbulent and thermally stable, and use of the "normal" values of C_T and k will then lead to excessively high rates of cooling. Also the generation of turbulent energy by surface roughness and shear is neglected in (3), so that its validity may be questionable in conditions of strong winds or strong shear across the top of the mixed layer. If the above caveats can be ignored, however, and the radiative flux profile is known, Eqs. (1)–(3) provide a complete system for calculation of the turbulent flux profile and rate of heating or cooling of the mixed layer.

Fig. 1 shows schematic profiles of radiative and turbulent heat fluxes for cases when the surface buoyancy flux is either positive (Fig. 1a) or negative (Fig. 1b) and with $k \approx 0.3$. The solid curves correspond to the Kahn-Businger assumption, with radiative flux decreasing exponentially downward from the value F_+ at cloud top. The dashed curves are drawn in agreement with the L assumption, and the dotted curves correspond to the S assumption, with surface turbulent heat flux and F_+ maintained the same for all three models. It is evident that the total (radiative plus turbulent) mixed-layer

TABLE 1. Summary of assumptions and equations used in three "dry cloud" models.

Equation	Lilly discontinuous flux (L) model	Schubert (S) model	Kahn-Businger (KB) model
Radiative flux assumption	$F = \begin{cases} 0, z < H \\ F_+, z > H \end{cases}$	$F = \begin{cases} F_0 + \mu(F_+ - F_0)\dfrac{z}{H}, z < H \\ F_+, z > H \end{cases}$ $F_0 = F_+ e^{-H/\lambda}$	$F = \begin{cases} F_+ e^{(z-H)/\lambda}, z < H \\ F_+, z > H \end{cases}$
(A) Mean change	$\dfrac{\partial\bar{\theta}}{\partial t} = [(\overline{w'\theta'})_0 - (\overline{w'\theta'})_-]/H$	$\dfrac{\partial\bar{\theta}}{\partial t} = [(\overline{w'\theta'})_0 - (\overline{w'\theta'})_- + \mu(F_0 - F_+)]/H$	$\dfrac{\partial\bar{\theta}}{\partial t} = [(\overline{w'\theta'})_0 - (\overline{w'\theta'})_- + F_0 - F_+]/H$
(B) Surface turbulent flux	$(\overline{w'\theta'})_0 = C_T V(\theta_s - \bar{\theta})$	same	same
(C) Cloud-top turbulent flux	$(\overline{w'\theta'})_- = \begin{cases} -k^{-1}(\overline{w'\theta'})_0, (\overline{w'\theta'})_0 < 0 \\ -k(\overline{w'\theta'})_0, \quad (\overline{w'\theta'})_0 > 0 \end{cases}$	same	$\left[1 - (1 - k^2)\left(\dfrac{z_2^2 - z_1^2}{H^2}\right)\right][(\overline{w'\theta'})_- + F_+]$ $= -\left[1 - (1 - k^2)\left(\dfrac{z_2 - z_1}{H}\right)\left(2 - \dfrac{z_2 + z_1}{H}\right)\right]$ $\times [(\overline{w'\theta'})_0 + F_0] + [1 - e^{-H/\lambda} - (1 - k^2)$ $\times (e^{(z_2-H)/\lambda} - e^{(z_1-H)/\lambda})]\dfrac{2\lambda}{H}F_+$
(D) Levels of zero turbulent flux	$z_1 = \dfrac{kH}{1+k}$ for $(\overline{w'\theta'})_0 < 0$ $z_2 = \dfrac{H}{1+k}$ for $(\overline{w'\theta'})_0 > 0$	same	$\left(1 - \dfrac{z_1}{H}\right)[(\overline{w'\theta'})_0 + F_0] + \dfrac{z_1}{H}[(\overline{w'\theta'})_- + F_+]$ $- F_+ e^{(z_1-H)/\lambda} = 0$ for $(\overline{w'\theta'})_0 < 0$ $\left(1 - \dfrac{z_2}{H}\right)[(\overline{w'\theta'})_0 + F_0] + \dfrac{z_2}{H}[(\overline{w'\theta'})_- + F_+]$ $- F_+ e^{(z_2-H)/\lambda} = 0$
(E) Mixed-layer growth	$\dfrac{\partial H}{\partial t} = -DH + \dfrac{F_+ - (\overline{w'\theta'})_-}{\theta_{u0} + H d\theta/dz - \bar{\theta}}$	$\dfrac{\partial H}{\partial t} = -DH + \dfrac{(1 - \mu)(F_+ - F_0) - (\overline{w'\theta'})_-}{\theta_{u0} + H d\theta/dz - \bar{\theta}}$	$\dfrac{\partial H}{\partial t} = -DH - \dfrac{(\overline{w'\theta'})_-}{\theta_{u0} + H d\theta/dz - \bar{\theta}}$

heating rates are smaller and cooling rates larger for both the KB and S assumptions than for the L model. The results of calculations from Deardorff's model and Randall's model should appear generally similar to those of KB, except that in the Deardorff model the discontinuity in turbulent heat flux at the cloud top is smoothed out over the upper transition region, while in the Randall model the fluxes are piecewise linear rather than exponential.

Distributed radiative cooling also exerts an effect on the entrainment rate at the cloud top and the rate of change of mixed-layer depth. The heat balance for the entire mixed layer can be written as

$$H\frac{\partial\bar{\theta}}{\partial t} = \left(\frac{\partial H}{\partial t} - w_H\right)\Delta(\theta) + (\overline{w'\theta'})_0 - (F_+ - F_0), \quad (4)$$

where w_H is the mean vertical velocity at cloud top and Δ denotes the difference of quantities across the cloud top interface (upper minus lower). Thus for a given surface turbulent flux and net radiative loss, the decreased mixed-layer heating rates predicted by the KB and S models must be reflected in reduced entrainment of warm air. Elimination of $\partial\bar{\theta}/\partial t$ between (4) and (1), written at $z = H$, yields the interface condition derived more directly by Lilly, i.e.,

$$\left(\frac{\partial H}{\partial t} - w_H\right)\Delta(\theta) = \Delta(\overline{w'\theta'} + F). \quad (5)$$

Since it is assumed that turbulent fluxes vanish above the mixed layer, the first term on the right of (5) is simply $-(\overline{w'\theta'})_-$, the turbulent flux just below cloud top. The second term vanishes for the KB assumption, equals F_+ for L, and is assumed to be a fraction of $F_+ - F_0$ by S. Evaluation of these terms from Fig. 1 indicates, as expected, that the L model has the highest entrainment rate.

In order to show more quantitatively the effects of distributed radiative cooling, we have carried out calculations for both transient and steady-state mixed layers using the three models discussed above. Table 1 summarizes the assumptions and equations used in our calculations. In the prescription of radiative flux, the ratio of distributed flux to the total is μ in the S model. In the KB model the flux decreases exponentially away from the cloud top, with an extinction length scale λ. Note that the prescription of F_0 in the S model is such as to assure that it has the same net radiative cooling as that of KB. Eqs. (A)–(C) and (E) of Table 1 are either identical to or derived from (1)–(3) and (5) of the text, with the first of these evaluated at or just below cloud top. Eq. (D) evaluates the levels z_1 (lower) and z_2 (upper) at which buoyancy flux reverses sign. At least one of these must exist for $k > 0$. Eqs. (C) and (D) are much more complicated for the KB exponential flux model than for the others, requiring iterative solution and demonstrating the practical attractiveness of the S model.

TABLE 2. Entrainment rates for various ratios of surface heat flux to radiative flux.

	$(\partial H/\partial t - w_H)\Delta(\theta)/F_+$		
$(\overline{w'\theta'})_0/F_+$	-0.18	0	10
L model	0.1	1	3
S model	$0.1 - \mu$	$1 - \mu$	$3 - \mu$
KB model	~ 0	0.37	2.5

Eq. (E) varies between models according to the magnitude of the radiative flux jump assumed across the cloud top. The denominator of the second term on the right is obtained from the assumption that above the mixed layer the potential temperature increases linearly with height, i.e., $\theta = \theta_{u0} + z\,d\theta/dz$. The first term on the right of (E) is derived on the assumption that D, the large scale divergence, is constant with height, so that the mean vertical velocity at cloud top is linearly proportional to H. This is perhaps a reasonable assumption in equatorward trade wind flow, where the divergence is associated with the β term of the vertical vorticity equation. In an anticyclonically curved flow situation, on the other hand, a mixed-layer model assumption for momentum would suggest that the subsidence at the top of the layer is independent of its depth.

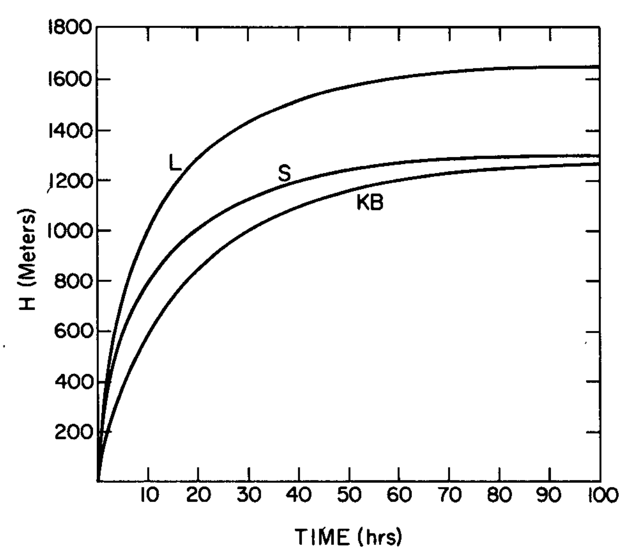

FIG. 2. Time evolution of mixed-layer depth for the Lilly discontinuous flux (L), Kahn-Businger (KB) for Schubert (S) model, as described in the text.

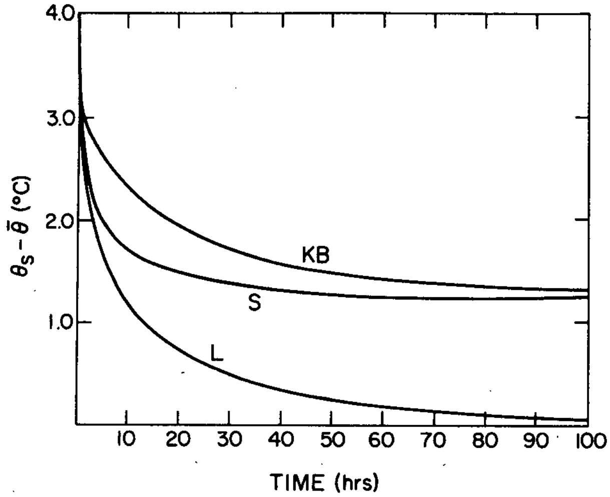

FIG. 3. Time evolution of the surface-air temperature difference for the three models defined in previous caption.

A model breakdown arises in the case of strong negative surface heat flux. Consideration of Eqs. (C), (D) and (E) shows that the L model predicts negative entrainment ($\partial H/\partial t + DH < 0$) if $(\overline{w'\theta'})_0 < kF_+$, with a similar result for the other models at somewhat smaller negative surface fluxes. This prediction is inconsistent with the existence of turbulence in the mixed layer. It is clearly a failure of the energy balance assumption [Eq. (3)], but a large enough negative surface-air temperature difference to cause such an effect might also lead to questions of the adequacy of the transfer coefficient expression [Eqs. (2) or (B)]. In any case it can only be a very transient situation, since the rapid cooling

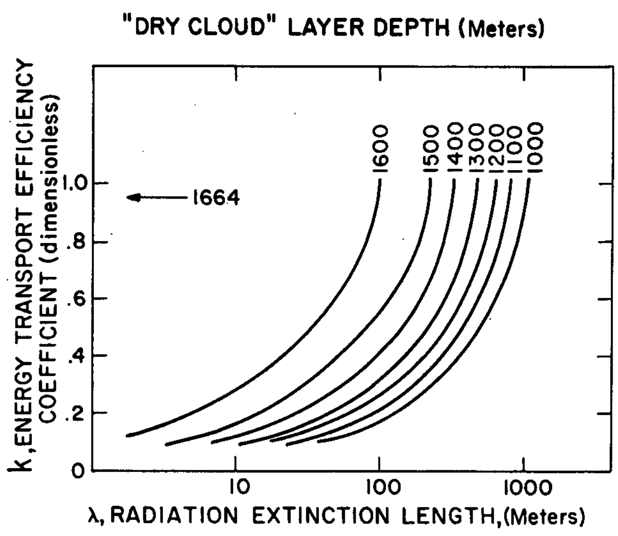

FIG. 4. Steady-state solutions for mixed-layer depth in meters for the Kahn-Businger model as a function of the radiation extinction length λ and of the energy transport efficiency coefficient k. The value 1664 m is for the case $\lambda = 0$, $k > 0$, corresponding to the Lilly discontinuous flux model.

from both radiative and surface transfer will soon reduce or eliminate the negative surface heat flux.

An indication of the difference between the model assumptions can be obtained by a comparison of entrainment rates. Table 2 provides such a comparison for various ratios of the surface heat flux to the radiative flux at cloud top, where the entrainment rates are made dimensionless by multiplication by $\Delta(\theta)/F_+$. For the KB model the radiative length scale λ is specified to be $0.1\,H$ and in each case $k = 0.2$. The parameter μ of the Schubert model is left unspecified. The table shows that for all models the entrainment is small or negative at $(\overline{w'\theta'})_0/F_+ = -0.18$. For large positive surface heat flux all models approach the non-radiative result, for which the entrainment rate is given by $\partial H/\partial t - w_H = k(\overline{w'\theta'})_0/\Delta(\theta)$. The largest difference occurs at and near zero surface heat flux, where the Lilly model predicts almost three times as large an entrainment rate as that of Kahn and Businger, with the Schubert model somewhat intermediate, depending on the chosen value of μ.

We carried out time-dependent calculations for growing mixed layers using the following parameters, taken from Lilly (1968) and Schubert *et al.* (1979):

$$\left.\begin{aligned} &C_T V = 0.015\ \mathrm{m\ s^{-1}}, \quad \theta_s - \theta_{u0} = 3^\circ\mathrm{C}, \\ &D = 5.5 \times 10^{-6}\ \mathrm{s^{-1}} \\ &F_+ = 0.06033^\circ\mathrm{C\ m\ s^{-1}}, \\ &d\theta/dz = 5.76 \times 10^{-3}\ ^\circ\mathrm{C\ m^{-1}} \end{aligned}\right\}.$$

The energy balance parameter k is set equal to 0.2. In the S model the ratio of distributed to total radiation flux μ is set equal to $e^{-1} \approx 0.37$, and in the KB model the extinction length scale λ is specified to be 50 m. The initial boundary-layer depth is 50 m, and the initial mixed-layer temperature is 4°C cooler than the surface.

Figs. 2 and 3 show the evolution of the three models for the first 100 h, when steady state is closely approached for each model. The steady-state solutions for the two distributed radiative cooling cases are nearly the same for the coefficients chosen. Those cases differ from the L model in having positive surface heat flux and reduced mixed-layer depth, which is consistent with the reduced entrainment rate expected from previous reasoning. On the other hand, the initial transient behavior of The S model more closely resembles the L result, in that the mixed-layer depth and potential temperature grow about twice as fast as they do for the KB model. Referring to (A) of Table 1, we see that the increase in θ is produced entirely by turbulent flux divergence for the L case, is reduced by radiative flux divergence for the S model, and is more substantially reduced in the KB model. Similarly (E) shows that the contributions to mixed layers growth are maximized for the L and minimized for the KB models.

We have not carried out similar transient calculations for initially negative surface-air temperature difference. From consideration of (A) and (E) it would appear that cooling of the mixed layer temperature would be greatest for the KB and least for the L model. The growth of the mixed-layer depth would generally be negative if subsidence were significant, and the S mixed layer should grow slower or shrink faster than that of the L model.

We have also investigated the sensitivity of the steady-state solutions of the three models to the distributed radiation amounts and the energy transport efficiency. The previously listed parameters are used in all cases, but k and either λ or μ are varied. Plots of mixed layer depth for the KB model are shown in Fig. 4, as a function of k and λ, and for the S model in Fig. 5 as a function of k and μ. The layer depth for the L model is 1664 m, independent of k. Clearly, the L model is the correct limit of the KB model as the radiation extinction length λ vanishes, except perhaps at $k = 0$. It is evident that the sensitivity of the models to distributed radiative cooling is greatest for small assumed values of k, the energy transport efficiency. For $k = 0.2$ an assumed radiative extinction length of only 10 m produces a 10% decrease in steady-state layer depth for the KB model, whereas $\lambda = 125$ m decreases the layer depth by ~⅓. Evidently, also, the S model can be "tuned" to adjusting μ to produce the same steady-state layer depth as that of the KB model for given values of λ and k. It can be shown from Eqs. (A), (B) and (E) that if the external parameters $C_T V$, D, $\theta_s - \theta_{u0}$, $d\theta/dZ$ and F_+ are fixed and if the S and KB models give the same H, then they also give the same surface-air temperature difference $\theta_s - \bar{\theta}$. Thus, not only is an isopleth of H also an isopleth of $\theta_s - \bar{\theta}$ (as in Fig. 5), but the value of $\theta_s - \bar{\theta}$ associated with a given H is valid for both the S and KB models.

In conclusion, we have shown that for the "dry cloud" case, the effects of distributed radiative cooling are moderately significant for both steady-state and transient solutions. The principal effect is to reduce the entrainment rate from that calculated from the L model. Steady-state solutions show decreases in the height of the cloud layer (when divergence is assumed constant with height) and the mean potential temperature. Transient solutions with distributed cooling tend to cool faster and warm slower, again from the decreased entrainment rate. The S linear flux approximation appears to give results similar to that of KB if time scales of environmental variations are relatively slow, so that conditions are fairly close to steady state, but its transient behavior far from steady state is more like that of the L model.

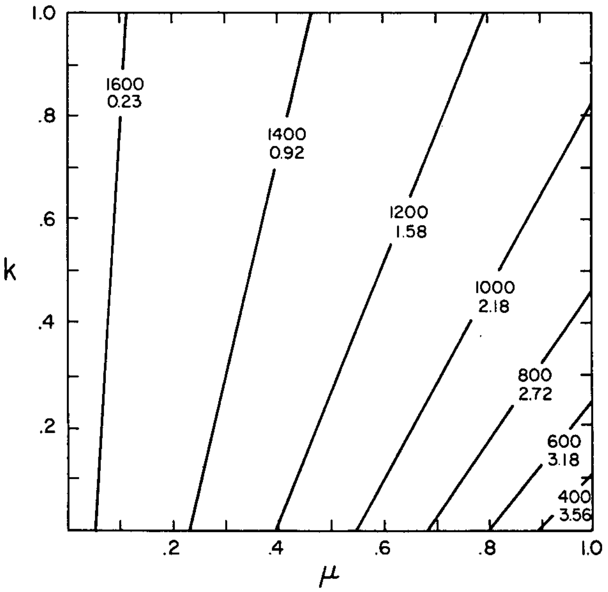

FIG. 5. Steady-state solutions for mixed-layer depth in meters and surface-air temperature difference (°C) for the Schubert model as a function of the ratio of distributed to total radiative flux divergence μ and of k.

Extension of the analysis to the wet cloud case remains incomplete, although Schubert *et al.* (1979) and Randall (1980) have both carried out extensive steady-state calculations using S-type and piecewise linear type models. One can reason that the reduction in entrainment rate would tend to both cool and moisten the mixed layer, leading to lowering of both cloud tops and bases. This qualitative behavior seems to be a principal result of the abovementioned calculations, although the details are somewhat sensitive to the temperature of the cloud layer.

REFERENCES

Deardorff, J. W., 1976: On the entrainment rate of a stratocumulus-topped mixed layer. *Quart. J. Roy. Meteor. Soc.*, **102**, 563–582.

——, 1979: Stratocumulus-topped mixed layers derived from a three-dimensional model. Submitted to *Bound.-Layer Meteor.*

——, and J. A. Businger, 1980: Comments on "Marine stratocumulus convection, Part I: Governing equations and horizontally homogeneous solutions." *J. Atmos. Sci.*, **37**, 481–482.

Kahn, P. H., and J. A. Businger, 1979: The effect of radiative flux divergence on entrainment of a saturated convective boundary layer. *Quart. J. Roy. Meteor. Soc.*, **105**, 303–306.

Kraus, H., and E. Schaller, 1978: A note on the closure in Lilly-type inversion models. *Tellus*, **30**, 284–288.

Lilly, D. K., 1968: Models of cloud-topped mixed layers under a strong inversion. *Quart. J. Roy. Meteor. Soc.*, **94**, 292–309.

Randall, D. A., 1980: Entrainment into a stratocumulus layer with distributed radiative cooling. *J. Atmos. Sci.*, **37**, $$$–$$$.

Schubert, W. H., J. S. Wakefield, E. J. Steiner and S. K. Cox, 1979: Marine stratocumulus convection, Part I: Governing equations and horizontally homogeneous solutions. *J. Atmos. Sci.*, **36**, 1286–1307.