A Spectral Cumulus Parameterization for Use in Numerical Models of the Tropical Atmosphere

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ABSTRACT

The spectral cumulus parameterization theory of Arakawa and Schubert is presented in the convective flux form as opposed to the original detrainment form. This flux form is more convenient for use in numerical prediction models. The equations are grouped into one of three categories that are members of a control flow diagram: feedback, static control, and dynamic control. The dynamic control, which determines the cloud base mass flux distribution, is formulated as an optimization problem. This allows quasi-equilibrium to be satisfied as closely as possible while maintaining the necessary nonnegativity constraint on the cloud base mass flux.

Results of two applications of the parameterization are shown. The first illustrates the dependence of the predicted cloud mass flux distribution on the vertical profile of the large-scale vertical motion field. According to the assumption of quasi-equilibrium of the cloud work function, the mass flux associated with deep clouds is controlled by large-scale vertical motion in the middle and upper troposphere, not just by vertical motion at the top of the mixed layer. The second application shows the evolution of the mass flux distribution during the simulated intensification of a tropical vortex using an axisymmetric primitive equation model. A similar sensitivity of deep convection to the development of upper level vertical motion is also observed. These examples demonstrate the inherent potential of this spectral approach for helping to establish a better understanding of the physical nature of the interaction of organized cumulus convection with the large-scale fields not available in more conventional empirical parameterization methods.

1. Introduction

Early linear stability analyses of the growth of small amplitude perturbations in a conditionally unstable environment were unable to explain the observed size and growth rates of the tropical cyclone (e.g., Syono, 1953; Lilly, 1960). This apparent failure of theory led Charney and Eliassen (1964) and Ooyama (1964) to introduce the concept of Conditional Instability of the Second Kind (CISK) which embodies a cooperative interaction between the cumulus scale and large scale. In a broad sense, CISK describes a situation in which the large-scale circulation is responsible for organizing and maintaining cumulus convection by providing the necessary horizontal transport of water vapor, while the cumulus-scale drives the large-scale circulation through the release of latent heat in deep convective elements. Both Charney and Eliassen and Ooyama dealt with the large-scale explicitly, but treated the convective-scale implicitly, i.e., the cumulus activity was specified to be a function of the large-scale fields, or was treated by what is now commonly referred to as cumulus parameterization.

Charney and Elliasen's and Ooyama's separate treatment of the large- and convective-scale motions stimulated efforts to numerically simulate the tropical atmosphere using a variety of convective parameterization techniques. The cumulus parameterizations employed in these models were highly empirical, neglecting many of the physical processes involved in the mutual interaction of cloud and environment. These included schemes in which the convective-scale heating rates were dependent upon the large-scale convergence of water vapor in the atmospheric boundary layer (Ooyama, 1969; Ogura, 1964), and the net large-scale convergence of water vapor throughout the depth of the troposphere (Kuo, 1965), as well as the more familiar moist convective adjustment (Manabe *et al.*, 1965). Although such schemes have performed remarkably well in numerical integrations, their relatively crude nature limits their ability to contribute to a greater understanding of the interaction between cumulus clouds and the larger-scale tropical circulations. Unfortunately, suitable alternatives were unavailable for many years, primarily because of a lack of knowledge regarding the interaction of cumulus clouds with the larger scale.

The problem of establishing the physical nature of the interaction of organized cumulus convection with the large-scale fields is fundamental to tropical meteorology. A clear understanding of this interaction is in all likelihood essential to an understanding of the dynamics of tropical phenomena in general. In recent years, many diagnostic (and some prognostic) studies have been made that have led to an improvement in our knowledge of cumulus convection, and consequently to an improvement in cumulus parameterization theory. Simple conceptual cloud models have proven to be useful in diagnosing the interaction of precipitating and nonprecipitating cumulus ensembles with the larger-scale motions (e.g., Yanai *et al.*, 1973; Ogura and Cho, 1973; Gray, 1973; Betts, 1975; Nitta, 1975, 1977, 1978; Yanai *et al.*, 1976; Johnson, 1976, 1977). These and other studies have helped to establish a general consensus on how cumulus convection modifies and maintains the large-scale thermodynamic fields.

Ooyama (1971) recognized the need to improve cumulus parameterization theory, and was the first to propose a theory taking into account the coexistence of a spectrum of clouds. The clouds were represented by independent entraining buoyant elements dispatched from the mixed layer. The theory was not closed, however, since the determination of the *dispatcher function* was left to future consideration.

Arakawa and Schubert (1974) have proposed a cumulus parameterization theory that describes the mutual interaction of an ensemble of cumulus clouds with the large-scale environment. The cloud field (or cloud ensemble) is represented by a spectrum of idealized model clouds (sub-ensembles) each of which has its own mass, heat and moisture budget. The vertical transports accomplished by this spectrum of model clouds is ultimately determined by the cloud base mass flux for each member of the spectrum. In order to determine this mass flux distribution, Arakawa and Schubert propose the concept of quasi-equilibrium of the cloud work function, which leads to an integral equation relating the cloud base mass flux distribution to large-scale thermodynamic processes. Observational support for the quasi-equilibrium assumption has been presented by Lord and Arakawa (1980), and a semi-prognostic test of the overall theory using GATE data is presented in Lord (1982). Although this is perhaps the most physically complete approach to the problem of cumulus parameterization proposed to date, experience with its use in linear (e.g., Stark, 1976; Wada, 1977; Crum and Stevens, 1983) and nonlinear models has only recently begun to accumulate.

In this paper we shall present the theory in a form more appropriate for use in numerical prediction models. The relationship between this flux formulation and the original detrainment formulation, as well as modifications made for computational reasons, will also be discussed. From a computational point of view, the most difficult aspect of the application of this cumulus parameterization is the solution of the integral equation for the cloud base mass flux distribution. The conventional techniques suggested for solving this equation are deficient since they do not guarantee a nonnegative mass flux distribution, a necessary constraint if the solution is to be considered physically reasonable. By relaxing the quasi-equilibrium assumption, however, it is possible to formulate an optimization problem that constrains the cloud base mass flux to be nonnegative. This procedure, which is referred to as the optimal adjustment method, is discussed in Section 2c. Finally, in Section 3 we shall present some examples of the behavior of the parameterization as applied in a semi-prognostic study, and in an axisymmetric primitive equation simulation of a tropical disturbance.

2. Governing equations

The mutual interaction between the cloud ensemble and the large-scale environment is conceptually illustrated in Fig. 1 where the equations of the theory have been grouped into three categories: feedback, static control, and dynamic control (Schubert, 1974). The equations that constitute the feedback part of the loop describe how the cumulus-scale transport terms, and source/sink terms modify the large-scale thermodynamic fields, while the equations comprising the static and dynamic control parts of the interaction loop describe how the properties of the cloud ensemble are controlled by the large-scale fields.. We continue our discussion of the parameterization theory within this framework.

a. Feedback

The complete theory divides the large-scale environment into a subcloud mixed layer of variable depth and the region of cumulus convection above the mixed layer, separated by an infinitesimally thin transition layer (see Fig. 2). In the subcloud mixed layer, the dry static energy s ($s \equiv c_pT + \phi$), water vapor mixing ratio q, and therefore the moist static energy h ($h \equiv s + Lq$), are constant with height and are denoted by the symbols s_M, q_M, and h_M. The top of the subcloud mixed layer p_B is generally somewhat below cloud base p_C. Below p_B, convective-scale transports are accomplished by the turbulence of the mixed layer, where the turbulence is confined below p_B by the stable and infinitesimally thin transition layer. Across the transition layer there can be discontinuities in the dry static energy and moisture, as well as discontinuities in the

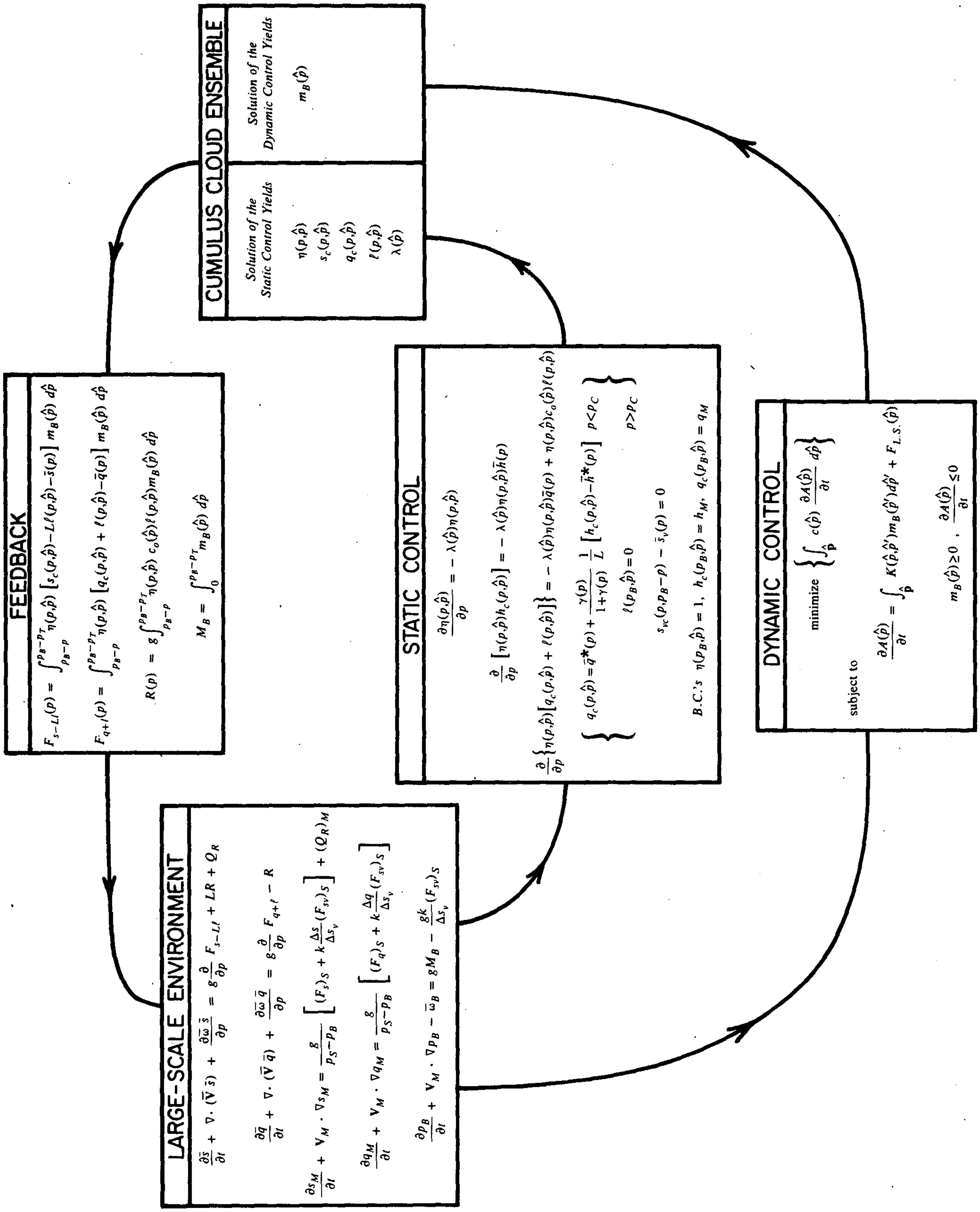

FIG. 1. Schematic representation of cloud environment interaction.

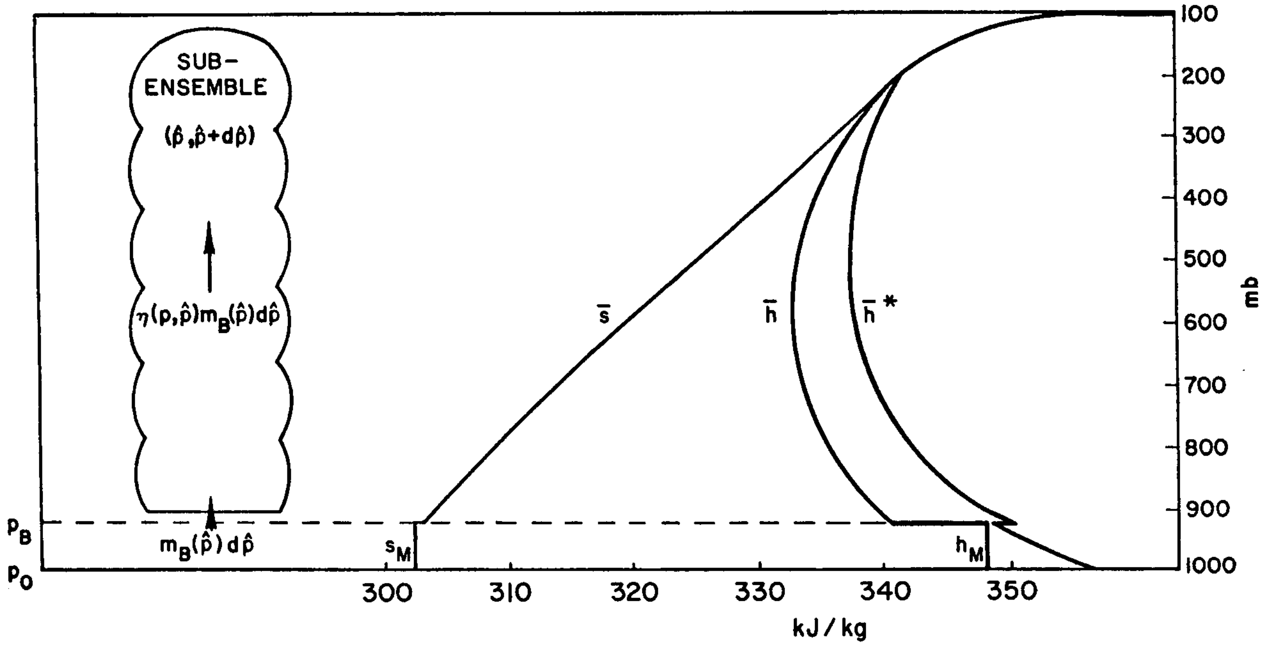

FIG. 2. Typical ITCZ profiles of $\bar{s}$, $\bar{h}$ and $\bar{h}^*$. Above p_B these profiles are those of Yanai *et al.* (1973). The schematic subensemble has cloud base p_C slightly above p_B. The mass flux at level p is $\eta(p, \hat{p})m_B(\hat{p})d\hat{p}$ while the mass flux at p_B is $m_B(\hat{p})d\hat{p}$.

convective-scale fluxes. Above p_B, the convective-scale transports are accomplished by the cloud ensemble. Let us write the heat and moisture budget equations for this region in terms of dry static energy, and water vapor mixing ratio. These are

$$\frac{\partial \bar{s}}{\partial t} + \nabla\cdot(\bar{\mathbf{V}}\bar{s}) + \frac{\partial \bar{\omega}\bar{s}}{\partial p} = g\frac{\partial}{\partial p}F_{s-Ll} + LR + Q_R, \quad (2.1)$$

$$\frac{\partial \bar{q}}{\partial t} + \nabla\cdot(\bar{\mathbf{V}}\bar{q}) + \frac{\partial \bar{\omega}\bar{q}}{\partial p} = g\frac{\partial}{\partial p}F_{q+l} - R. \quad (2.2)$$

The barred quantities represent horizontal averages over an area large enough to contain an ensemble of clouds, but small enough so as to cover only a fraction of a large-scale disturbance. The convective-scale fluxes of liquid water static energy, $s - Ll$, and total water, $q + l$, are given by

$$\left.\begin{aligned} F_{s-Ll} &\equiv F_s - LF_l, \\ F_{q+l} &\equiv F_q + F_l, \end{aligned}\right\} \quad (2.3)$$

where the fluxes of dry static energy, water vapor and liquid water are defined by

$$F_s(p) = \int_{p_B-p}^{p_B-p_T} \eta(p, \hat{p})[s_c(p, \hat{p}) - \bar{s}(p)]m_B(\hat{p})d\hat{p}, \quad p \leqslant p_B, \quad (2.4)$$

$$F_q(p) = \int_{p_B-p}^{p_B-p_T} \eta(p, \hat{p})[q_c(p, \hat{p}) - \bar{q}(p)]m_B(\hat{p})d\hat{p}, \quad p \leqslant p_B, \quad (2.5)$$

$$F_l(p) = \int_{p_B-p}^{p_B-p_T} \eta(p, \hat{p})l(p, \hat{p})m_B(\hat{p})d\hat{p}, \quad p \leqslant p_B. \quad (2.6)$$

The convective-scale liquid water sink R is defined by

$$R(p) \equiv g\int_{p_B-p}^{p_B-p_T} \eta(p, \hat{p})c_0(\hat{p})l(p, \hat{p})m_B(\hat{p})d\hat{p}. \quad (2.7)$$

We see from (2.4)–(2.7) that the cumulus cloud ensemble has been spectrally divided into *sub-ensembles* each of which is characterized by its pressure depth $\hat{p} \equiv p_B - p_D$, where p_D is the detrainment pressure level. Our use of $\hat{p}$ as the spectral parameter differs from the original formulation in which the sub-ensembles were characterized by the fractional entrainment rate λ. This alteration is motivated by computational convenience and will be discussed further when we consider the dynamic control part of the theory. Thus, the dry static energy, water vapor, and liquid water for sub-ensemble $\hat{p}$ at level p are respectively denoted by $s_c(p, \hat{p})$, $q_c(p, \hat{p})$, and $l(p, \hat{p})$. The vertical mass flux at level p due to sub-ensemble $\hat{p}$ is $\eta(p, \hat{p})m_B(\hat{p})d\hat{p}$ where $\eta(p, \hat{p})$ is the normalized mass flux which has unit value at the top of the subcloud mixed layer p_B. A simple physical interpretation of (2.4)–(2.6) is that for each sub-ensemble $\hat{p}$, the net upward flux at level p of a particular quantity (such as s) is given by the difference between the upward flux of that quantity inside the sub-ensemble (denoted by subscript c) and the downward flux of the environmental value (denoted by a bar). This downward mass flux in the environment is merely the compensating subsidence produced by the sub-ensemble. Since the environment does not contain liquid water there is no downward flux of liquid water due to environmental subsidence, and the convective-scale liquid water flux takes a simpler form. The total ensemble flux at level p of any quantity (such as F_s) is then given by an integral over all sub-ensembles that penetrate level p.

The expression for R states that the quantity of water removed from the atmosphere at level p by sub-ensemble $\hat{p}$ is simply proportional to the sub-ensemble liquid water content at that level. As originally formulated, Arakawa and Schubert chose an autoconversion coefficient, c_0, which was independent of sub-ensemble. However, Silva Dias and Schubert (1977) used the results of a theoretical parametric model of cumulus convection (Lopez, 1973) to demonstrate that such an approach probably underestimated the precipitation associated with deep clouds while overestimating the precipitation associated with shallow clouds. Thus, we use an autoconversion coefficient that is dependent upon $\hat{p}$ such that deep convection is more efficient than shallow convection at producing precipitation. We have drawn the autoconversion coefficient, in units of m^{-1}, as a function of $\hat{p}$ in Fig. 3. For the purpose of interpreting the diagram, the reader should be reminded that the autoconversion coefficient used in our pressure formulation is related to the original autoconversion coefficient by the factor $\bar{H}/p$, where $\bar{H}$ is simply the scale height, $R\bar{T}/g$. The total ensemble precipitation rate P is obtained by integrating $R(p)$ over the region above the mixed layer, i.e.,

$$P = \frac{1}{g}\int_{p_T}^{p_B} R(p)dp. \tag{2.8}$$

Below p_B the convective-scale fluxes of s and q are linear in pressure with values of $(F_s)_S$, and $(F_q)_S$ at the surface, and $(F_s)_B$, and $(F_q)_B$ just below p_B. The convective-scale flux of l is zero everywhere below p_B. The governing equations for this region are derived from the heat and moisture budgets for the subcloud mixed layer of variable depth as well as for the transition layer. Since the transition layer is assumed to be infinitesimally thin, the heat and moisture budgets for this layer turn out to be conditions on the discontinuities across the layer. The budget equations for the dry static energy and moisture of the mixed layer (s_M, and q_M respectively), and the governing equation for the time dependence of p_B, are given by

$$\frac{\partial s_M}{\partial t} + \mathbf{V}_M\cdot\nabla s_M = \frac{g}{p_S - p_B}\left[(F_s)_S + k\frac{\Delta s}{\Delta s_v}(F_{sv})_S\right] + (Q_R)_M, \tag{2.9}$$

$$\frac{\partial q_M}{\partial t} + \mathbf{V}_M\cdot\nabla q_M = \frac{g}{p_S - p_B}\left[(F_q)_S + k\frac{\Delta q}{\Delta s_v}(F_{sv})_S\right], \tag{2.10}$$

$$\frac{\partial p_B}{\partial t} + \mathbf{V}_M\cdot\nabla p_B - \bar{\omega}_B = gM_B - \frac{gk}{\Delta s_v}(F_{sv})_S. \tag{2.11}$$

The quantity $(F_{sv})_S$ is the flux of virtual dry static energy at the surface and k is an empirical constant which relates this surface flux to its value just below p_B. In (2.9)–(2.11) the symbol Δ represents the jump of the particular property across the top of the subcloud mixed layer p_B (e.g., $\Delta s \equiv \bar{s}(p_{B-}) - \bar{s}_M$). The quantity

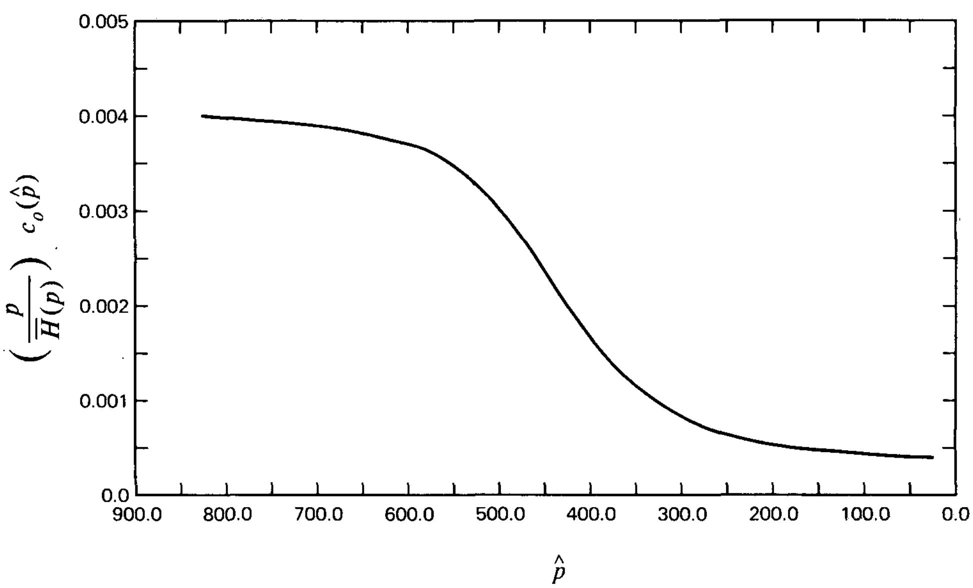

FIG. 3. The autoconversion coefficient c_0 (m^{-1}), as a function of $\hat{p}$.

M_B is the total cloud base mass flux associated with the cumulus ensemble, i.e.,

$$M_B = M_C(p_B), \tag{2.12}$$

where

$$M_C(p) = \int_{p_B-p}^{p_B-p_T} \eta(p, \hat{p}) m_B(\hat{p}) d\hat{p}, \quad p \leq p_B. \tag{2.13}$$

The cumulus ensemble transport terms, F_{s-Ll}, F_{q+l}, M_B, and the cumulus ensemble liquid water sink R constitute the feedback part of the interaction loop shown in Fig. 1. From (2.4)–(2.7), and (2.13) we see that the determination of these quantities is equivalent to the determination of $\eta(p, \hat{p})$, $s_c(p, \hat{p})$, $q_c(p, \hat{p})$, $l(p, \hat{p})$, and $m_B(\hat{p})$. All except $m_B(\hat{p})$ are determined in the static control part of the interaction loop while $m_B(\hat{p})$ is determined by the dynamic control. Once these quantities are known, it is possible to predict the time variation of the temperature and moisture field above p_B as well as the time dependent behavior of p_B.

b. *Static control*

The sub-ensemble normalized mass flux, moist static energy and total water content are determined from their respective budget equations. For the very simple cloud model we have chosen, they are given by

$$\frac{\partial \eta(p, \hat{p})}{\partial p} = -\lambda(\hat{p})\eta(p, \hat{p}), \tag{2.14}$$

$$\frac{\partial}{\partial p}[\eta(p, \hat{p})h_c(p, \hat{p})] = -\lambda(\hat{p})\eta(p, \hat{p})\bar{h}(p), \tag{2.15}$$

$$\frac{\partial}{\partial p}\{\eta(p, \hat{p})[q_c(p, \hat{p}) + l(p, \hat{p})]\}$$
$$= -\lambda(\hat{p})\eta(p, \hat{p})\bar{q}(p) + \eta(p, \hat{p})c_0(\hat{p})l(p, \hat{p}), \tag{2.16}$$

where the fractional entrainment rate $\lambda(\hat{p})$ has the units Pa^{-1}. The air inside the sub-ensembles is assumed to be saturated at a temperature only slightly different from the environmental temperature, an assumption that gives rise to the saturation relation

$$q_c(p, \hat{p}) = \bar{q}^*(p)$$
$$+ \frac{\gamma(p)}{1 + \gamma(p)} \frac{1}{L}[h_c(p, \hat{p}) - \bar{h}^*(p)], \tag{2.17}$$

where $\bar{q}^*(p)$ is the saturation value of q at level p, $\bar{h}^*(p)$ is the saturated moist static energy at level p and $\gamma(p) \equiv (L/c_p)[\partial\bar{q}^*/\partial\bar{T}]_p$ (c.f. Arakawa, 1969). In order to determine the individual sub-ensemble budgets, knowledge of the fractional entrainment rate $\lambda(\hat{p})$ is required. This entrainment rate is given implicitly by the vanishing buoyancy condition

$$s_{vc}(p, p_B - p) - \bar{s}_v(p) = 0, \tag{2.18}$$

or using the definition of the virtual dry static energy[1]

$$s_c(p, p_B - p) - \bar{s}(p)$$
$$+ c_p\bar{T}(p)\delta[q_c(p, p_B - p) - \bar{q}(p)] = 0, \tag{2.19}$$

where $\delta = 0.608$.

Thus, the static control part of the interaction loop consists of the five equations (2.14)–(2.17) and (2.19) in the five unknown variables $\eta(p, \hat{p})$, $s_c(p, \hat{p})$, $q_c(p, \hat{p})$, $l(p, \hat{p})$, and $\lambda(\hat{p})$. Since (2.14)–(2.16) are differential equations which are solved upward from p_B, they require the appropriate boundary conditions which are $\eta(p_B, \hat{p}) = 1$, $h_c(p_B, \hat{p}) = h_M$, $q_c(p_B, \hat{p}) = q_M$, and $l(p_B, \hat{p}) = 0$. Discrete analogues of these equations (including numerical algorithms for solving the vanishing buoyancy condition) are discussed in Silva Dias and Schubert (1977) and Lord *et al.* (1982).

c. *Dynamic control and the optimal adjustment method*

The last remaining problem is the determination of the mass flux distribution function $m_B(\hat{p})$ since once it is known, the time variation of the temperature and moisture fields, and subcloud mixed layer top p_B can be predicted from (2.1), (2.2), and (2.11). In order to determine $m_B(\hat{p})$, Arakawa and Schubert first introduce the *cloud work function*

$$A(\hat{p}) = \int_{p_B-\hat{p}}^{p_B} \eta(p, \hat{p})[s_{vc}(p, \hat{p}) - \bar{s}_v(p)] \frac{dp}{p}, \tag{2.20}$$

an integral measure of the buoyancy force associated with sub-ensemble $\hat{p}$, with the weighting function $\eta(p, \hat{p})$. Physically, $A(\hat{p}) > 0$ can be thought of as a generalized criterion for moist convective instability, while $A(\hat{p}) \leq 0$ is indicative of a neutral or stable situation. Since the variables in the integrand of (2.20) are either prognostic variables, or are related diagnostically to prognostic variables, the time rate of change of $A(\hat{p})$ can be written in terms of the time derivatives of s_M, q_M, p_B, $\bar{s}(p)$, and $\bar{q}(p)$ (we hereafter refer to barred and mixed layer quantities as *large-scale* quantities). These time derivatives are in turn related to two types of terms: convective-scale terms which are proportional to the cloud base mass flux distribution $m_B(\hat{p})$, and the large-scale terms which include horizontal and vertical advection, radiation, and surface energy exchanges. Thus, the time rate of change of $A(\hat{p})$ can be expressed as the sum of convectively induced changes, and large-scale changes, or

$$\frac{\partial A(\hat{p})}{\partial t} = \left(\frac{\partial A(\hat{p})}{\partial t}\right)_{\mathrm{CS}} + \left(\frac{\partial A(\hat{p})}{\partial t}\right)_{\mathrm{LS}}. \tag{2.21}$$

Since the convective-scale terms depend linearly on $m_B(\hat{p})$ and all subensembles participate in determining $(\partial A(\hat{p})/\partial t)_{\mathrm{CS}}$, it can be shown that

[1] The effects of liquid water on buoyancy have been neglected.

$$\frac{\partial A(\hat{p})}{\partial t} = \int_0^{p_B - p_T} K(\hat{p}, \hat{p}')m_B(\hat{p}')d\hat{p}' + F(\hat{p}), \quad (2.22)$$

where the kernel $K(\hat{p}, \hat{p}')$ and the large-scale forcing $F(\hat{p})$ are known. The kernel represents either a destruction or generation of $A(\hat{p})$ by sub-ensemble $\hat{p}'$ if sub-ensemble $\hat{p}'$ has unit cloud base mass flux.

Arakawa and Schubert proposed a closure hypothesis, referred to as *quasi-equilibrium,* which requires balance between the large-scale generation of $A(\hat{p})$ and the convective-scale destruction of $A(\hat{p})$ for all sub-ensembles. Mathematically, this closure hypothesis takes the form

$$\int_0^{p_B - p_T} K(\hat{p}, \hat{p}')m_B(\hat{p}')d\hat{p}' + F(\hat{p}) = 0. \quad (2.23)$$

It is appropriate at this point to consider the use of $\hat{p}$ as the spectral parameter, rather than λ (fractional entrainment rate) as in the original theory. Because the parameterization will be incorporated in a vertically discrete model atmosphere, the use of λ as the spectral parameter can introduce computational difficulties. In order to follow sub-ensemble λ in time, the detrainment pressure level $p_D(\lambda)$ would become a function of time. Since the vertical coordinate is fixed for all time at a finite number of points, λ could be retained only with interpolation of the cloud work function in λ space which could introduce large errors in the application of the theory. Thus we have chosen $\hat{p}$, the cloud depth pressure, as the spectral parameter since the calculation of $\partial A(\hat{p})/\partial t$ poses much less of a computational problem in a model that has pressure (or transformed pressure) as the vertical coordinate.

We note that this change in the spectral parameter somewhat alters quasi-equilibrium as originally formulated, since $(\partial A(\hat{p})/\partial t)_{CS} \neq (\partial A(\lambda)/\partial t)_{CS}$. The use of $\hat{p}$ gives rise to a second term which contributes to a modification of the kernel characteristics as discussed by Schubert (1973). This term involves the time rate of change of the fractional entrainment rate of sub-ensemble $\hat{p}$, since λ is now an independent variable. The selection of the spectral parameter is one of the arbitrary aspects of the cloud model. As one example, Lord and Arakawa (1980) have assembled observational evidence that shows the cloud work function to be a quasi-universal function of detrainment level p_D.[2] Our choice of $\hat{p}$ is motivated purely by computational convenience, and although this choice modifies the quantitative details of the interaction of members of the cumulus ensemble, the qualitative aspects of this interaction remain unchanged.

Requiring balance between the large-scale generation and convective-scale destruction of $A(\hat{p})$ means that the equation for $m_B(\hat{p})$ takes the form of a Fredholm integral equation of the first kind. The various schemes suggested for solving this type of equation do not guarantee a nonnegative mass flux distribution which is a necessary constraint if the solution is to be regarded as physically reasonable. In order to avoid the difficulties associated with obtaining negative cloud base mass fluxes in the solution, we have chosen to restate the quasi-equilibrium hypothesis as an optimization problem (Hack and Schubert, 1976) which can be written in the following form.

Let $\hat{\mathbf{P}}$ represent the subset of the $\hat{p}$ domain for which $F(\hat{p}) > 0$. We wish then to

$$\text{minimize} \left\{ \int_{\hat{\mathbf{P}}} c(\hat{p}) \frac{\partial A(\hat{p})}{\partial t} d\hat{p} \right\},$$

subject to

$$\frac{\partial A(\hat{p})}{\partial t} = \int_{\hat{\mathbf{P}}} K(\hat{p}, \hat{p}')m_B(\hat{p}')d\hat{p}' + F(\hat{p})$$

for

$$m_B(\hat{p}) \geq 0, \quad \frac{\partial A(\hat{p})}{\partial t} \leq 0. \quad (2.24)$$

Formulating the problem in this way requires quasi-equilibrium to be satisfied as closely as possible while constraining the cloud base mass flux distribution to be nonnegative. This particular formulation of the problem is referred to as the *over-adjustment* case by Silva Dias and Schubert (1977) who have investigated other formulations of the optimization problem (e.g., the *under-adjustment* case). In (2.24), both $\partial A(\hat{p})/\partial t$ and $m_B(\hat{p})$ are regarded as unknowns, while $c(\hat{p})$, $K(\hat{p}, \hat{p})$ and $F(\hat{p})$ are regarded as knowns. The weighting function $c(\hat{p})$ is defined to be negative in order to maintain a mathematically well posed minimization problem. The discrete form of (2.24) turns out to be a linear programming problem which is readily solved using the simplex method (Dantzig, 1963; Luenberger, 1973) and is discussed in the following paragraphs as the optimal adjustment method.

As we saw in Section 2a, the processes that contribute to changes in the large-scale temperature and moisture fields can be divided into two parts: large-scale terms, and convective-scale terms. In a numerical model, these terms are often computed separately using different time steps, sometimes differing by an order of magnitude or more. Thus, it is convenient from a computational point of view to formulate the cumulus parameterization in terms of an adjustment process. Although we have used the word *adjustment,* our procedure should not be confused with the moist convective adjustment methods used in many numerical models. The adjustment process we shall discuss is purely a consequence of the time discretization associated with the numerical model.

Let us define the atmosphere to be stable to sub-

[2] In the absence of variations in the top of the top of the subcloud layer, our spectral parameter reduces to sub-ensemble detrainment pressure level discussed by Lord and Arakawa (1980).

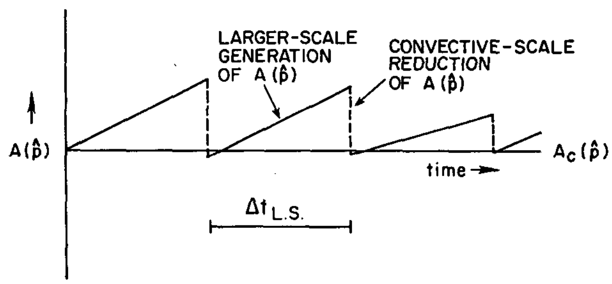

FIG. 4. Schematic representation of the *over-adjustment* case of the optimal adjustment method.

ensemble $\hat{p}$ if the cloud work function $A(\hat{p})$ is smaller than some critical value $A_c(\hat{p})$. Thus, the atmosphere is considered to be respectively neutral or unstable to each sub-ensemble depending on whether $A(\hat{p})$ equals or exceeds this critical value. If the large-scale terms push the atmosphere into an unstable state, it is the job of the dynamic control (2.24) to determine a mass flux distribution that will adjust the atmosphere back at least to (but at the same time as close as possible to) the neutral state for each $\hat{p}$ subject to the constraint that each sub-ensemble mass flux be nonnegative (see Fig. 4).

Suppose we have n cloud types (where a *cloud type* is the discrete analogue of *sub-ensemble*). Let m_i be the cloud base mass flux of the ith cloud type and f_i be the amount that the ith cloud work function exceeds the neutral (or critical) value ($f_i > 0$). If cloud type j contributes an amount K_{ij} per unit mass flux to the reduction of f_i [where K_{ij} is the discrete analogue of the kernel $K(\hat{p}, \hat{p}')$], we can write

$$\left.\begin{aligned}
&K_{11}m_1 + K_{12}m_2 + \cdots + K_{1n}m_n \geqslant f_1,\\
&K_{21}m_1 + K_{22}m_2 + \cdots + K_{2n}m_n \geqslant f_2,\\
&\quad\vdots \qquad\quad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots\\
&K_{n1}m_1 + K_{n2}m_2 + \cdots + K_{nn}m_n \geqslant f_n,\\
&\qquad m_1 \geqslant 0,\ m_2 \geqslant 0 \cdots m_n \geqslant 0.
\end{aligned}\right\} \tag{2.25}$$

Eq. (2.25) states that an adjustment greater than or equal to f_i must occur for each i and that each sub-ensemble (cloud type) mass flux must be nonnegative.

Each inequality (2.25) can be converted to an equality by introducing a *surplus variable x*. For inequality i, the surplus variable x_i represents the surplus adjustment made to work function i. Thus, inequality i takes the form

$$K_{i1}m_1 + K_{i2}m_2 + \cdots + K_{in}m_n - x_i = f_i. \tag{2.26}$$

The objective is to minimize some measure of the surplus adjustment. Assuming that this measure is linear and gross in character, we can write

$$\left.\begin{aligned}
&\qquad\qquad \text{minimize } \{\sum_{i=1}^{n} c_i x_i\}\\
&\text{subject to}\\
&K_{11}m_1 + K_{12}m_2 + \cdots + K_{1n}m_n - x_1 = f_1\\
&K_{21}m_1 + K_{22}m_2 + \cdots + K_{2n}m_n - x_2 = f_2\\
&\quad\vdots \qquad\quad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \qquad \vdots\\
&K_{n1}m_1 + K_{n2}m_2 + \cdots + K_{nn}m_n - x_n = f_n,\\
&\qquad m_1 \geqslant 0,\quad m_2 \geqslant 0 \cdots m_n \geqslant 0,\\
&\qquad x_1 \geqslant 0,\quad x_2 \geqslant 0 \cdots x_n \geqslant 0,
\end{aligned}\right\} \tag{2.27}$$

where the c_i are the weights. In more compact vector notation, we can write

$$\left.\begin{aligned}
&\qquad\qquad \text{minimize } \mathbf{c}\cdot\mathbf{x},\\
&\text{subject to}\\
&\qquad \mathbf{Km} - \mathbf{x} = \mathbf{f},\quad \mathbf{m} \geqslant 0,\quad \mathbf{x} \geqslant 0.
\end{aligned}\right\} \tag{2.28}$$

Thus we have one minimization objective, n adjustment constraints and $2n$ nonnegativity constraints. Solution of the problem yields the n unknown sub-ensemble mass fluxes, the n unknown surplus adjustments, and the value of our objective function, $\mathbf{c}\cdot\mathbf{x}$, which is a gross measure of the surplus adjustment. The optimization problem as formulated is easily solved using the simplex method of linear programming. From a computational point of view, however, this over-adjustment formulation can be inconvenient since the simplex procedure requires a basic feasible solution with which to start. A simple reinterpretation of the optimization problem allows the formulation of the under-adjustment case where the initial basic feasible solution required by the simplex procedure, is trivially given by *slack variables*.

Silva Dias and Schubert (1977) have studied the sensitivity of the linear programming problem to the selection of the weighting function (or cost vector) **c** and have determined that although there is only a weak dependence on **c**, slightly more reasonable results are obtained when using $c_i = f_i^{-1}$ (i.e. making the weighting function inversely proportional to the desired adjustment)[3]. This is the procedure we follow in this study.

d. Relationship between the flux and detrainment forms of the equations

The forms of the equations for $\bar{s}(p)$ and $\bar{q}(p)$ given by (2.1) and (2.2) are the convective-scale *flux* forms. Since they differ in appearance from the *detrainment* forms presented in Arakawa and Schubert (1974) we wish to say a few words about their relationship. The detrainment forms can easily be derived by differentiating (2.4)–(2.6) with respect to p and making use of the definitions given in (2.3) and the sub-ensemble

[3] A physical argument for determining **c** could perhaps come from a consideration of duality theory.

budget equations of the static control. The two forms are mathematically equivalent, but the convective-scale flux forms have advantages when used in models that are discrete in the vertical.

3. Results

In this section we illustrate the behavior of the parameterization in a one-dimensional *semi-prognostic* application as well as in a nonlinear axisymmetric primitive equation model. By *semi-prognostic* we mean that we evaluate instantaneous tendencies produced by the cumulus parameterization at a single grid point in response to a large-scale forcing, the computation in effect terminated after only one time step. The results from the primitive equation model involve many hundreds of time steps, and allow the predicted cloud field to evolve in time in response to the large-scale fields and to the modifications made to the large-scale fields by the action of the clouds themselves.

We note that for computational simplicity we do not include a subcloud mixed layer of variable depth in the present experiments, but rather a mixed layer whose top is defined by a fixed coordinate surface. This simplification is motivated by the difficulties associated with the vertical difference representation of such a layer. Such difficulties can be remedied, for example, by employing a generalized vertical coordinate where the top of the mixed layer is a coordinate surface that is also a function of time. For now, our results are confined to the simpler situation in which the mixed layer is fixed by the vertical coordinate. This modification allows the cumulus convection to directly influence the energy budget of the mixed layer, rather than determine the depth of the mixed layer as in the more general theory. Consequently, in the above formulation, the convective-scale fluxes of s and q are continuous across the top of the subcloud mixed layer even though the large-scale values are not. One additional approximation we make is that the cloud base p_C and the top of the model mixed layer p_B are one and the same.

a. Semi-prognostic application

In this section we examine the response of cumulus convection to the destabilization of the atmosphere by large-scale processes. We define response to be the predicted cloud base mass flux and associated precipitation for a given large-scale forcing. For the experiments presented here, the only difference in the forcing is the vertical profile of the large-scale vertical motion. The large-scale temperature and moisture profiles used are those obtained by Yanai *et al.* (1973) from their analysis of the Marshall Islands observations. A horizontally homogeneous thermodynamic field is assumed as is the climatological radiative cooling profile of Dopplick (1970). Constant surface heat and moisture fluxes of 11 W m^{-2} and 130 W m^{-2} are also assumed. Three typical large-scale vertical motion profiles, $\bar{M}$ ($\bar{M} = -\bar{\omega}/g$), are considered in this study and are shown in Fig. 5. The upper tropospheric peaked profile is characteristic of active regions in the Western Pacific, while the double peaked profile is more representative of GATE disturbed periods. The third profile, which exhibits a lower tropospheric peak, was also observed

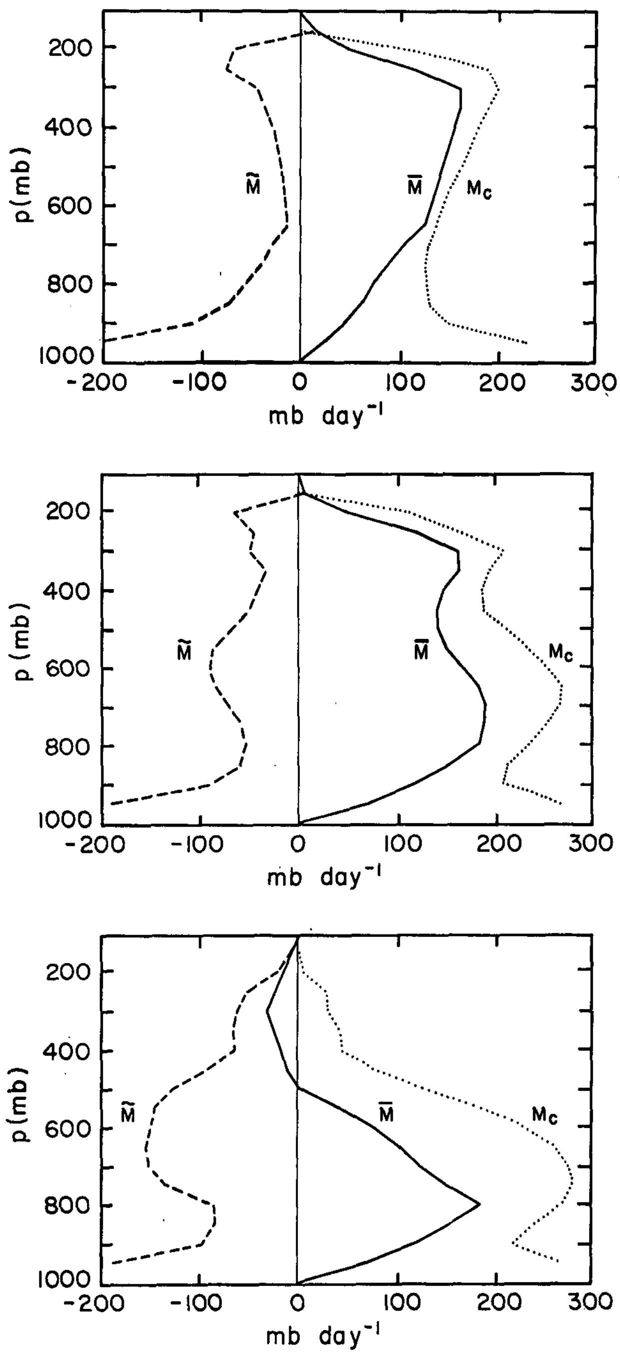

FIG. 5. The large-scale vertical motion $\bar{M}$ (solid line), the total cloud mass flux M_C obtained by the quasi-equilibrium assumption (dotted line), and the residual mass flux $\tilde{M}$ (dashed line).

during GATE but occurs during the cluster dissipating stage. The magnitude of each $\bar{M}$ was chosen so that the double peaked profile would share the same values of the upper tropospheric peaked profile between 100 mb and 350 mb, and of the lower tropospheric peaked profile between 1000 mb and 800 mb.

The total cloud ensemble mass flux M_C (see Eq. 2.13) predicted by the cumulus parameterization is also shown in Fig. 5 for each of the large-scale vertical motion profiles as is the residual or environmental mass flux $\tilde{M}$ ($\tilde{M} = \bar{M} - M_C$). The upper tropospheric peaked $\bar{M}$ exhibits a highly bimodal behavior of convective mass flux (supporting both shallow and deep convection) while the double peaked profile produces a pronounced enhancement of middle level convection. It is worth noting that although a 2:1 difference in $\bar{M}$ is observed at cloud base, the resulting upper level M_C is remarkably similar. This result suggests that the existence of deep clouds is not well correlated with the large-scale vertical motion in low levels. Yanai *et al.* (1976) in a diagnostic study (independent of the quasi-equilibrium assumption) also observed that deep convection was better correlated with large-scale upper level vertical motion than with lower tropospheric peaks in $\bar{M}$. Additional support for such a conclusion can be found in the third case (lower tropospheric peaked $\bar{M}$) we have examined. The predicted M_C indicates a moderate enhancement of shallow convection with respect to the double peaked forcing, and little cloud mass flux at high levels. Thus, although we see the same low level $\bar{M}$ in each of these cases, the existence of deep convection depends on the forcing in the upper levels of the troposphere. Furthermore, since deep convection is more efficient at producing precipitation, we should expect to see significant differences in the rainfall rate. These rates are given in Table 1, and show a nearly 2:1 difference for the double and lower tropospheric peaked vertical motion profiles, despite the fact that the profiles are identical below 800 mb. A similar, although smaller ratio is observed with respect to the upper tropospheric peaked vertical motion profile.

One final observation of the results presented in Fig. 5 is that when deep convection is suppressed, the cloud mass flux associated with shallow clouds tends to be enhanced. Although this effect is not large, it is most noticeable in the 700–850 mb layer. Yanai *et al.* (1976) have also found a very weak and negative correlation between $\bar{M}$ and M_C near 750 mb.

b. *Application in a primitive equation model*

In this section we present some of the cumulus parameterization results obtained from a numerical simulation of a weak axisymmetric tropical disturbance. The governing equations consist of the primitive equations formulated for an f-plane using axisymmetric cylindrical coordinates in the horizontal and the sigma coordinate in the vertical. Space and time differencing schemes follow those developed by Arakawa for the UCLA GCM with a vertical resolution of approximately 50 mb (18 levels) and a horizontal resolution of 15 km. The model also employs a lateral radiation boundary condition at 960 km (Hack and Schubert, 1981) which minimizes the reflection of gravity–inertia waves generated in the interior of the computational domain. Additional detail regarding the design of this model can be found in Hack and Schubert (1980).

TABLE 1. Predicted precipitation rates for $\bar{M}$ profiles shown in Fig. 5.

	Upper tropospheric peaked $\bar{M}$	Double peaked $\bar{M}$	Lower tropospheric peaked $\bar{M}$
Precipitation rate (cm/day)	2.44	3.01	1.54

In addition to the vertical transport of heat and moisture, we allow the cumulus ensemble to vertically transport (i.e., redistribute) horizontal momentum. A budget equation, which is similar to those for heat and moisture above p_B can be derived for the horizontal momentum $\mathbf{V}$ and is written

$$\frac{d\bar{\mathbf{V}}}{dt} + f\mathbf{k} \times \bar{\mathbf{V}} + \nabla\bar{\phi} = g\frac{\partial}{\partial p}F_{\mathbf{V}}, \qquad (3.1)$$

where the convective-scale flux of momentum is defined as

$$F_{\mathbf{V}}(p) = \int_{p_B-p}^{p_B-p_T} \eta(p, \hat{p})[\mathbf{V}_c(p, \hat{p}) - \bar{\mathbf{V}}(p)]m_B(\hat{p})d\hat{p},$$

$$p \leq p_B. \qquad (3.2)$$

The sub-ensemble horizontal momentum $\mathbf{V}_c(p, \hat{p})$ must be determined as a function of the large-scale fields, a difficult problem since $\mathbf{V}_c(p, \hat{p})$ is not a conserved quantity as are some thermodynamic properties. However, here we follow the simple alternative (e.g., Ooyama, 1971; Arakawa *et al.*, 1974; Schneider and Lindzen, 1976) of assuming that $\mathbf{V}_c$ is conservative which leads to the following subensemble budget equation for momentum

$$\frac{\partial}{\partial p}[\eta(p, \hat{p})\mathbf{V}_c(p, \hat{p})] = -\lambda(\hat{p})\eta(p, \hat{p})\bar{\mathbf{V}}(p), \quad (3.3)$$

where the lower boundary condition is $\mathbf{V}_c(p_B, \hat{p}) = \mathbf{V}_M$.

We assume that initially the mass and tangential wind fields are in gradient wind balance and that there is no transverse circulation. The initial tangential wind field consists of a broad circulation with a maximum value of just under 7 m s^{-1} at 240 km. During the first 24 h of the numerical integration, the model disturbance begins to develop a deep radial circulation. The early time evolution of the cloud base mass flux distribution (see Fig. 6) shows the rapid development of deep cumulus convection in the interior of the computational domain. Six hours into the numerical simulation the cloud population is primarily composed of shallow clouds with some very weak deep convection around 200 km radius. The large-scale forcing of cumulus convection at this point consists of surface eddy fluxes of sensible heat and moisture coupled with a weak low-level convergence. As the simulation proceeds, a bimodal distribution of cloud-base mass flux evolves and begins to move very slowly inward. The development of deep convection follows a pronounced

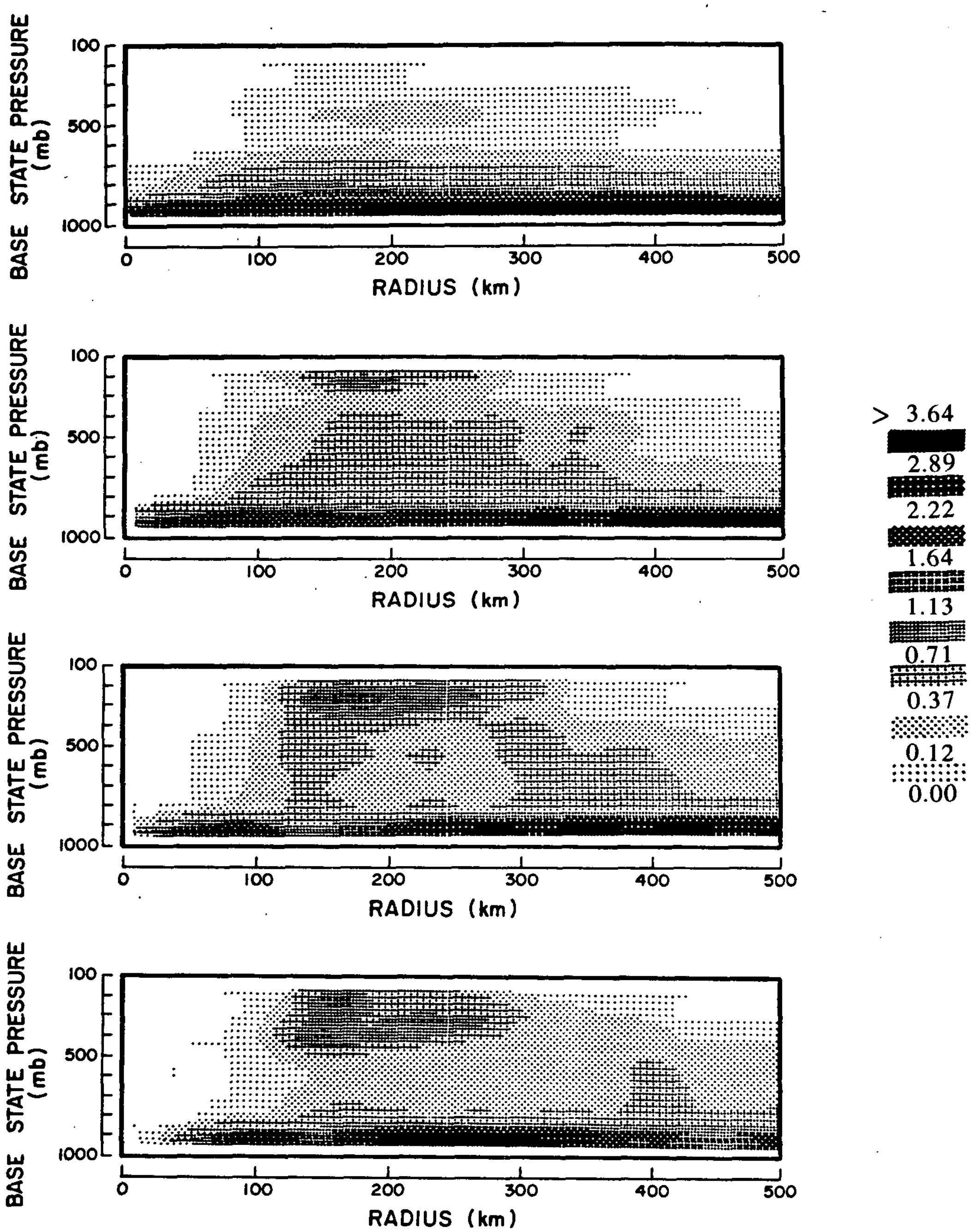

FIG. 6. The cloud base mass flux distribution (mb h^{-1}) at 6, 12, 18 and 24 h as a function of radius for a two-dimensional primitive equation simulation of a weak tropical disturbance. The shading at level p is proportional to the cloud base mass flux of the subensemble that detrains at level p.

moistening of the middle levels of the atmosphere by shallow and middle level clouds, and occurs together with the rapid development of an upper-level vertical motion field at around 200 km radius. As in the semi-prognostic study presented earlier, the existence of an upper-level vertical motion field appears to be essential to the establishment and maintenance of deep convection since, throughout the 24 h period shown, the low-level vertical motion field remains relatively unchanged. The development of deep cumulus clouds also has a significant impact on the convective precipitation rates as was also shown in the previous section (see Fig. 7). At 6 h, the weak and shallow cloud population precipitates almost uniformly with a peak rate equal to 0.19 cm d^{-1}. The development of deep convection enhances the peak rate by a factor of almost 50 at 24 h.

By the end of the 24 h period shown, the simulated disturbance has increased its maximum tangential wind to nearly 10 m s^{-1} and an upper level anticyclonic circulation outside of 300 km has developed. This weak disturbance goes on to develop into an intense and well organized hurricane disturbance reaching a steady state by 96 h. However, the details of such simulations, which incorporate the spectral parameterization, will not be dealt with here but will be reported on in future work.

4. Concluding remarks

We have presented the Arakawa–Schubert spectral cumulus parameterization in a framework that conceptually divides the mutual interaction of the cumulus and large-scale into the categories of feedback, static control and dynamic control. The equations constituting the feedback part of the interaction loop describe how the cumulus-scale transport and source terms modify the large-scale temperature and moisture fields. The equations grouped in the static control category describe the normalized mass and energy budgets of each member of the cumulus ensemble. Finally, the dynamic control (the integral equation for $m_B(\hat{p})$) specifies how the large-scale fields control the total cloud ensemble mass flux and its distribution among the various sub-ensembles. The dynamic control is formulated in terms of an optimization problem that allows us to constrain the mass flux distribution to be nonnegative while satisfying quasi-equilibrium as closely as possible.

We are encouraged by the behavior of the above formulation of the spectral cumulus parameterization. The predicted cloud populations for both the semi-prognostic and primitive equation applications of the parameterization are quite reasonable, bearing considerable resemblance to those obtained in diagnostic budget studies (e.g., Yanai *et al.* 1976) that do not depend on the assumption of quasi-equilibrium. The parameterization predicts a highly interactive behavior between the large-scale fields and the cloud ensemble. In the examples presented, the large-scale forcing includes the effects of large-scale horizontal advection, surface sensible and latent heat flux, radiative cooling, and the dominating effect of the large-scale vertical motion. The semi-prognostic approach was designed to study the effects of the latter forcing, but the primitive equation simulation allows all of these processes to contribute to the destabilization of the atmosphere with respect to cumulus convection. As simple as they are, both applications demonstrate the importance of upper-level vertical motion to deep convection. They suggest that the correlation between low-level vertical motion and the existence of deep precipitating clouds is small, a conclusion reached independently using diagnostic methods.

Our study is primarily intended to demonstrate the tractability and potential advantages of the Arakawa–Schubert parameterization method. The cloud model that is described by the static control is of course highly idealized and should be improved so that it is more physically complete (e.g., Johnson, 1976; Nitta, 1978). With such improvements to the model, we hope the future use of this parameterization method in more sophisticated models of the tropical atmosphere can contribute to additional insight into the interactions

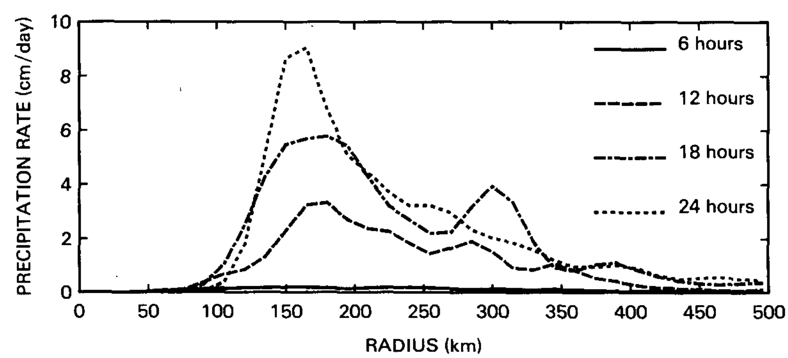

FIG. 7. Precipitation associated with the convection shown in Fig. 6.

between the cumulus-scale and large-scale that maintain the observed tropical circulations.

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