

Reply

THOMAS A. GUINN

Air Force Global Weather Central, Offutt Air Force Base, Nebraska

WAYNE H. SCHUBERT

Department of Atmospheric Science, Colorado State University, Fort Collins, Colorado

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We agree with Schade (1994) that the inclusion of diabatic and frictional processes and vertical structure is important for accurate simulation of hurricane spiral bands. Indeed, accurate simulations involve a complex interplay between advective, diabatic, and frictional processes. Our goal was not to simulate spiral bands with a "full physics" model. Rather, our goal in using a shallow-water model was to isolate some of the underlying advective mechanisms that are independent of the details of frictional and moist physical processes. These mechanisms include potential vorticity (PV) wave breaking, vortex merger, and the formation of vorticity pools and filaments during ITCZ breakdown by barotropic instability. Our results show that these fundamental mechanisms for banding are implicit in the nonlinear, slow manifold dynamics of the quasi-two-dimensional, dry dynamical core of such a three-dimensional full physics model. Thus, the shallow-water model simply provides a convenient framework in which to develop concepts about hurricane spiral bands. As we cautioned in the discussion of Fig. 8, spiral bands in our simplified model evolve as the model states move along a frictionless and adiabatic slow manifold, while nature's spiral bands apparently evolve along a frictional and diabatic slow manifold. While these two manifolds are quantitatively different, there is little conceptual difference. Thus, although inviscid, dry dynamical models are a long way from the complex moist tropical atmosphere, they can serve as a guide for the feasibility of certain arguments about the role of advective processes in hurricane bands. We hope researchers with "full physics" models will also explore some of these banding ideas. To provide observational tests of such ideas, we also hope that in the future more observational and analysis efforts will be directed toward the construction of fine-grain maps of

the potential vorticity field in hurricanes (Shapiro and Franklin 1994).

Another issue raised by Schade concerns the "type" of potential vorticity most useful in studying hurricane bands. To further explore this issue let us briefly review the derivation of the general potential vorticity principle. This derivation begins with the vector momentum equation

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{1}{2} \nabla(\mathbf{u} \cdot \mathbf{u}) - \mathbf{u} \times \boldsymbol{\zeta} = -\nabla\Phi - \alpha \nabla p + \mathbf{F}, \quad (1)$$

where \mathbf{u} is the three-dimensional velocity vector, $\boldsymbol{\zeta} = 2\boldsymbol{\Omega} + \nabla \times \mathbf{u}$ the absolute vorticity vector, Φ the geopotential, and \mathbf{F} the frictional force per unit mass. Taking the curl of (1) we obtain

$$\frac{\partial \boldsymbol{\zeta}}{\partial t} - \nabla \times (\mathbf{u} \times \boldsymbol{\zeta}) = \nabla p \times \nabla \alpha + \nabla \times \mathbf{F}. \quad (2)$$

Now consider an arbitrary scalar ψ . Scalar multiplication of (2) by $\nabla\psi$ yields

$$\begin{aligned} \nabla\psi \cdot \frac{\partial \boldsymbol{\zeta}}{\partial t} + \nabla \cdot [\nabla\psi \times (\mathbf{u} \times \boldsymbol{\zeta})] \\ = \nabla\psi \cdot (\nabla p \times \nabla \alpha) + (\nabla \times \mathbf{F}) \cdot \nabla\psi. \end{aligned} \quad (3)$$

Since $\nabla\psi \times (\mathbf{u} \times \boldsymbol{\zeta}) = \mathbf{u}(\boldsymbol{\zeta} \cdot \nabla\psi) - \boldsymbol{\zeta}(\mathbf{u} \cdot \nabla\psi)$ and $\mathbf{u} \cdot \nabla\psi = \dot{\psi} - \partial\psi/\partial t$, we can write $\nabla\psi \times (\mathbf{u} \times \boldsymbol{\zeta}) = \mathbf{u}(\boldsymbol{\zeta} \cdot \nabla\psi) + \boldsymbol{\zeta}(\partial\psi/\partial t - \dot{\psi})$. Taking the divergence of this last expression, we obtain

$$\begin{aligned} \nabla \cdot [\nabla\psi \times (\mathbf{u} \times \boldsymbol{\zeta})] &= \mathbf{u} \cdot \nabla(\boldsymbol{\zeta} \cdot \nabla\psi) \\ &+ (\boldsymbol{\zeta} \cdot \nabla\psi) \nabla \cdot \mathbf{u} + \boldsymbol{\zeta} \cdot \nabla \left(\frac{\partial\psi}{\partial t} - \dot{\psi} \right), \end{aligned} \quad (4)$$

since $\nabla \cdot \boldsymbol{\zeta} = 0$. Substitution of (4) into (3) results in

$$\begin{aligned} \frac{D}{Dt} (\boldsymbol{\zeta} \cdot \nabla\psi) + (\boldsymbol{\zeta} \cdot \nabla\psi) \nabla \cdot \mathbf{u} \\ = \boldsymbol{\zeta} \cdot \nabla\dot{\psi} + (\nabla \times \mathbf{F}) \cdot \nabla\psi + \nabla\psi \cdot (\nabla p \times \nabla \alpha). \end{aligned} \quad (5)$$

Corresponding author address: Dr. Wayne H. Schubert, Colorado State University, Fort Collins, CO 89523.

Using the continuity equation $D\alpha/Dt = \alpha\nabla \cdot \mathbf{u}$ to eliminate the divergence from (5), we obtain

$$\frac{D}{Dt}(\alpha\zeta \cdot \nabla\psi) = \alpha\zeta \cdot \nabla\dot{\psi} + \alpha(\nabla \times \mathbf{F}) \cdot \nabla\psi + \alpha\nabla\psi \cdot (\nabla p \times \nabla\alpha). \quad (6)$$

Now consider the two choices $\psi = \theta$ and $\psi = \theta_e^*$, where θ is the potential temperature and θ_e^* is the saturation equivalent potential temperature. Since θ and θ_e^* can be written as functions of p and α only, both choices lead to the disappearance of the $\alpha\nabla\psi \cdot (\nabla p \times \nabla\alpha)$ term¹ on the right-hand side of (6). Then, these two choices of ψ lead, respectively, to the dry potential vorticity principle

$$\frac{D}{Dt}(\alpha\zeta \cdot \nabla\theta) = \alpha\zeta \cdot \nabla\dot{\theta} + \alpha(\nabla \times \mathbf{F}) \cdot \nabla\theta, \quad (7)$$

and the saturation equivalent potential vorticity principle

$$\frac{D}{Dt}(\alpha\zeta \cdot \nabla\theta_e^*) = \alpha\zeta \cdot \nabla\dot{\theta}_e^* + \alpha(\nabla \times \mathbf{F}) \cdot \nabla\theta_e^*. \quad (8)$$

Which of these is more useful in studying hurricane bands? Schade argues in favor of (8). However, tropical cyclones contain both saturated and unsaturated regions. In saturated regions $\dot{\theta}_e^*$ vanishes and the saturation equivalent potential vorticity $\alpha\zeta \cdot \nabla\theta_e^*$ tends to be materially conserved (if frictional effects are secondary), while $\dot{\theta}$ does not vanish and the dry potential vorticity $\alpha\zeta \cdot \nabla\theta$ is not materially conserved. In unsaturated regions the situation is reversed; that is, $\dot{\theta}$ vanishes and the dry potential vorticity $\alpha\zeta \cdot \nabla\theta$ tends to be

materially conserved (again, if frictional effects are secondary), while $\dot{\theta}_e^*$ does not vanish and the saturation equivalent potential vorticity $\alpha\zeta \cdot \nabla\theta_e^*$ is not materially conserved. Note that Schade's argument in favor of (8) is based on the questionable conclusion that $\alpha\zeta \cdot \nabla\theta_e^*$ vanishes in both saturated and unsaturated regions.

We can summarize the above argument as follows. The combination of saturated and unsaturated regions found in tropical cyclones frustrates our search for a potential vorticity that approximately satisfies material conservation everywhere. Under such circumstances, for observational work or for diagnostic analysis of primitive equation model output, maps of both $\alpha\zeta \cdot \nabla\theta$ and $\alpha\zeta \cdot \nabla\theta_e^*$ can be produced to aid in the interpretation of the dynamics.

Although the above PV conservation arguments do not provide a basis for preferring either (7) or (8), there are certain contexts in which (7) is definitely preferable to (8). These involve balanced models in which some form of approximate potential vorticity is the single prognostic variable, and in which this variable must be inverted via a second-order partial differential equation to recover the associated balanced wind and mass fields (e.g., Schubert and Alworth 1987). In such cases the invertibility principle involving a balanced approximation to $\alpha\zeta \cdot \nabla\theta$ is simpler than the one involving a balanced approximation to $\alpha\zeta \cdot \nabla\theta_e^*$.

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¹ Note that the choice $\psi = \theta_e$, the equivalent potential temperature, does not lead to the disappearance of the $\alpha\nabla\psi \cdot (\nabla p \times \nabla\alpha)$ term.