Diabatic and frictional forcing effects on the intensity and structure of tropical cyclones

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Motivation for theoretical work

“In order to make the best attainable forecast of the future weather, it would be desirable to express the physical laws as exactly as possible, and determine the initial conditions as precisely as possible. Yet the ultimate achievement of producing perfect forecasts, by applying equations already known to be exact to initial conditions already known to be precise, if such a feat were possible, would not by itself increase our understanding of the atmosphere, no matter how important it might be from other considerations.”

Outline

• Basic tropical cyclone structure and cylindrical coordinates

• The intensity forecasting problem

• Diabatic forcing with applications of Eliassen’s balanced vortex model in the free atmosphere

• Shock-like structures in the tropical cyclone boundary layer due to frictional forcing

• Conclusions on the role of diabatic and frictional forcing

• Plans for creating a balanced model which includes frictional effects
Basic tropical cyclone structure and cylindrical coordinates
Visible imagery for Hurricane Earl on 1 Sept 2010
Underneath the clouds
Underneath the clouds
Underneath the clouds
Cylindrical Coordinates

Vorticity
\[ \zeta = \frac{\partial (r u)}{r \partial r} \]

Divergence
\[ \delta = \frac{\partial (r v)}{r \partial r} \]
The intensity forecasting problem
Atlantic Intensity Error Trends (1990-2012)

NHC Official Average Intensity Errors
Atlantic Basin Tropical Cyclones

From Cangialosi and Franklin (2013)
Atlantic Intensity Error for 2012 and 2007-2011

From Cangialosi and Franklin (2013)
Diabatic forcing with applications of Eliassen’s balanced vortex model in the free atmosphere
Why Evaluate Balanced Vortex Model Applications?

- Eliassen’s Balanced Vortex Model potentially offers a
  - simple
  - fast
  - elegant
  way to explain tropical cyclone intensity change through diabatic heating.
Assumptions for the Balanced Vortex Model

- Inviscid
- Axisymmetric
- Quasistatic
- Gradient Balanced
- Stratified
- $f$-plane
Balanced Vortex Model – Governing Equations

Gradient Balance: \[ (f + \frac{v}{r}) v = \frac{\partial \phi}{\partial r} \]  

Tangential Velocity: \[ \frac{\partial v}{\partial t} + u \left( f + \frac{\partial (rv)}{r \partial r} \right) + w \frac{\partial v}{\partial z} = 0 \]

Hydrostatic: \[ \frac{\partial \phi}{\partial z} = \frac{g}{T_0} T \]

Continuity: \[ \frac{\partial (ru)}{r \partial r} + \frac{\partial (\rho w)}{\rho \partial z} = 0 \]

Thermodynamic: \[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \left( \frac{\partial T}{\partial z} + \frac{RT}{c_p H} \right) = \frac{Q}{c_p} \]
Can solve for 1 of 2 PDEs

Transverse circulation

\[
\frac{\partial}{\partial r} \left( A \frac{\partial(r\psi)}{r \partial r} + B \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial z} \left( B \frac{\partial(r\psi)}{r \partial r} + C \frac{\partial \psi}{\partial z} \right) = \frac{g}{c_p T_0} \frac{\partial Q}{\partial r}
\]

Geopotential Tendency

\[
\frac{\partial}{r \partial r} \left( r \frac{A}{D} \frac{\partial \phi_t}{\partial r} + r \frac{B}{D} \frac{\partial \phi_t}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{B}{D} \frac{\partial \phi_t}{\partial r} + \frac{C}{D} \frac{\partial \phi_t}{\partial z} \right) = \frac{g}{c_p T_0} \left[ \frac{\partial}{r \partial r} \left( r \frac{B}{D} Q \right) + \frac{\partial}{\partial z} \left( \frac{C}{D} Q \right) \right]
\]
Can solve for 1 of 2 PDEs

Transverse circulation

\[
\frac{\partial}{\partial r} \left( A \frac{\partial (r\psi)}{r \partial r} + B \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial z} \left( B \frac{\partial (r\psi)}{r \partial r} + C \frac{\partial \psi}{\partial z} \right) = \frac{g}{c_p T_o} \frac{\partial Q}{\partial r}
\]

Geopotential Tendency

\[
\frac{\partial}{r \partial r} \left( r \frac{A}{D} \frac{\partial \phi_t}{\partial r} + r \frac{B}{D} \frac{\partial \phi_t}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{B}{D} \frac{\partial \phi_t}{\partial r} + \frac{C}{D} \frac{\partial \phi_t}{\partial z} \right) = \\
\frac{g}{c_p T_o} \left[ \frac{\partial}{r \partial r} \left( r \frac{B}{D} Q \right) + \frac{\partial}{\partial z} \left( \frac{C}{D} Q \right) \right]
\]
Static Stability

$$ \rho A = \frac{g}{T_0} \left( \frac{\partial T}{\partial z} + \frac{R_d T}{c_p H} \right) $$

Baroclinicity

$$ \rho B = -\frac{g}{T_0} \frac{\partial T}{\partial r} = - \left( f + \frac{2v}{r} \right) \frac{\partial v}{\partial z} $$

Inertial Stability

$$ \rho C = \left( f + \frac{2v}{r} \right) \left( f + \frac{\partial (rv)}{r \partial r} \right) $$

Elliptic Condition

$$ D = AC - B^2 > 0 $$
1-D Simplified Geopotential Tendency

2-D equation with zero Baroclinicity and constant static stability

\[ N^2 \frac{\partial}{r \partial r} \left( \frac{r}{f^2} \frac{\partial \phi_t}{\partial r} \right) + \left( \frac{\partial}{\partial z} - \frac{1}{H} \right) \frac{\partial \phi_t}{\partial z} = \frac{g}{c_p T_0} \left( \frac{\partial}{\partial z} - \frac{1}{H} \right) Q \]

1-D ordinary differential equation (radial structure only)

\[ \hat{T}_t - \frac{d}{r \partial r} \left( \ell^2 r \frac{d\hat{T}_t}{dr} \right) = \frac{\hat{Q}}{c_p} \]

\[ \frac{d\hat{T}_t}{dr} = 0 \quad \text{at} \ r = 0 \]

\[ \frac{d\hat{T}_t}{dr} = - \left( \frac{K_1(b/\ell_0)}{\ell_0 K_0(b/\ell_0)} \right) \hat{T}_t \quad \text{at} \ r = b \]

(\( ^\wedge \)) represents the radial part

Based on Musgrave et al. 2012
Idealized 1-D Balanced Solutions

![Graph showing tangential velocity, relative vorticity, and diabatic heating against radius from the center of the storm.]

- **Tangential Velocity [m s\(^{-1}\)]**
- **Relative Vorticity \([10^{-3} \text{ s}^{-1}]\)**
- **Diabatic Heating [K 6 hr\(^{-1}\)]**

Legend:
- Blue: Initial Tangential Velocity
- Red: Relative Vorticity
- Green: Predicted Tangential Velocity
- Gray: Diabatic Heating
Idealized 1-D Balanced Solutions

![Idealized 1-D Balanced Solutions Diagram](image)
Idealized 1-D Balanced Solutions

The graph illustrates the variation of different parameters with the radius from the center of the storm. The parameters include initial tangential velocity, predicted tangential velocity, relative vorticity, and diabatic heating. The graph shows how these parameters change at different radii, providing insights into storm dynamics.
Irene – 78 hr HWRF forecast on 18 UTC 21 Aug 2011
Lessons from 1-D Geopotential solutions

- Defines a powerful conceptual model for velocity tendency from
  - Diabatic heating
  - Radius of maximum wind
  - High inertial stability region

- In models and observed storms
  - Lacks vertical structure
  - Cannot dissipate storms
  - Resolution of diabatic heating too low
Can solve for 1 of 2 PDEs

### Transverse circulation

\[
\frac{\partial}{\partial r} \left( A \frac{\partial (r\psi)}{r \partial r} + B \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial z} \left( B \frac{\partial (r\psi)}{r \partial r} + C \frac{\partial \psi}{\partial z} \right) = \frac{g}{c_p T_0} \frac{\partial Q}{\partial r}
\]

### Geopotential Tendency

\[
\frac{\partial}{r \partial r} \left( r \frac{A}{D} \frac{\partial \phi_t}{\partial r} + r \frac{B}{D} \frac{\partial \phi_t}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{B}{D} \frac{\partial \phi_t}{\partial r} + \frac{C}{D} \frac{\partial \phi_t}{\partial z} \right) = \frac{g}{c_p T_0} \left[ \frac{\partial}{r \partial r} \left( r \frac{B}{D} Q \right) + \frac{\partial}{\partial z} \left( \frac{C}{D} Q \right) \right]
\]
2-D Balanced Solutions

Return to the transverse circulation

\[ \frac{\partial}{\partial r} \left( A \frac{\partial (r \psi)}{r \partial r} + B \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial z} \left( B \frac{\partial (r \psi)}{r \partial r} + C \frac{\partial \psi}{\partial z} \right) = \frac{g}{c_p T_0} \frac{\partial Q}{\partial r} \]

Boundary conditions

\[ \psi(r, 0) = \psi_0(r), \quad \psi(r, z_T) = 0, \]

\[ \psi(0, z) = 0, \quad \text{and} \]

\[ \frac{\partial \psi}{\partial r} = -\frac{\psi}{\ell} \quad \text{at} \quad r = r_B \]

Solve the transverse circulation via a successive over-relaxation iterative method

Include baroclinicity and non-constant static stability
Tangential velocity

Modified Rankine vortex with smoothing

\[ v(r, z) = \frac{1}{2} \zeta_0(z) \begin{cases} r & 0 \leq r \leq r_m(z) \\ r_m(z)^{\alpha + 1}(z)/r^{\alpha} & r_m(z) \leq r < \infty \end{cases} \]

\[ r_m(z) = \left( \frac{f}{f + \zeta_0(z)} \right)^{1/2} R_0 \]
Diabatic heating
Results - Streamfunction

Masters Defense Slocum
Results - Temperature Tendency

![Temperature Tendency Diagram](image)

- Master Defense Slocum
Results - Tangential Velocity Tendency

The graphs show the tangential velocity tendency as a function of radius and height, with color representing the magnitude of the velocity tendency. The upper graph uses units of m s\(^{-1}\) (6 hr\(^{-1}\)), while the lower graph uses units of K (6 hr\(^{-1}\)). The color bars indicate the range of values for each graph.
Shock-like structures in the tropical cyclone boundary layer due to frictional forcing
Hurricane Hugo (1989) – Track

Hurricane Hugo

- Position at 0000 UTC
- Position at 1200 UTC

Masters Defense Slocum
Hurricane Hugo (1989) – Radar

Hurricane Hugo Radar Reflectivity

- > 54
- 52 - 53
- 50 - 51
- 48 - 49
- 46 - 47
- 44 - 45
- 42 - 43
- 40 - 41
- 37 - 39
- 34 - 36
- 31 - 33
- 28 - 30
- 25 - 27
- 22 - 24
- < 22

Storm Position:
lat= 14.52N
lon= 54.52W

Time Composite
1710 - 1727 GMT

Domain: 60 x 60 km
Winds (mph) every 15 sec

NOAA AOML
Hurricane Research Division
Hurricane Hugo (1989) – Radial Profile

HUGO - 15SEP1989 - N42

- 1723-1729 UTC (434m)
- 1823-1829 UTC (2682m)

Radiation (km)
Inviscid Burgers’ Equation

- Model for nonlinear wave propagation: \( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0 \)
- Example initial condition: \( u(x, 0) = 1 - \cos(x) \)
Viscous Burgers’ Equation

- Now include a viscosity term: \( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \varepsilon \frac{\partial^2 u}{\partial x^2} \)
- Same initial condition: \( u(x, 0) = 1 - \cos(x) \)
Example of shocks forming and merging

Based on LeVeque 2002
Example of shocks forming and merging

$q(0)$ at time $t = 0.00000000$
Slab Boundary Layer Model

- Assume gradient balance
- Assume constant in time
- Assume radial velocity is zero
- Assume constant depth
- Solve for $u$ and $v$
Slab Boundary Layer Model - Governing Equations

Two predictive equations for the horizontal wind

\[
\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial r} - w^-(\frac{u}{h}) + \left( f + \frac{v + v_{gr}}{r} \right) (v - v_{gr}) - c_D U \frac{u}{h} + K \frac{\partial}{\partial r} \left( \frac{\partial (ru)}{r \partial r} \right)
\]

\[
\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial r} - w^- \left( \frac{v - v_{gr}}{h} \right) - \left( f + \frac{\partial (rv)}{r \partial r} \right) u - c_D U \frac{v}{h} + K \frac{\partial}{\partial r} \left( \frac{\partial (rv)}{r \partial r} \right)
\]

Two diagnostic equations for the vertical velocity

\[
w = -h \frac{\partial (ru)}{r \partial r} \quad \text{and} \quad w^- = \frac{1}{2} (|w| - w)
\]
Numerical results from Williams et al. 2013
Numerical results from Williams et al. 2013
Simple Analytical Model - Equations

Through simplifying the governing equations

\[
\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial r} - \frac{u}{\tau}
\]

\[
\frac{\partial v}{\partial t} = -u \left( f + \frac{\partial v}{\partial r} + \frac{v}{r} \right) - \frac{v}{\tau}
\]

where \( \tau \) = a “typical” value of \( h/(c_D U) \)

Analytical solutions

\[
u(r,t) = u_o(\hat{r}) e^{-t/\tau}
\]

\[
r v(r,t) = \hat{r} v_o(\hat{r}) e^{-t/\tau}
\]

Characteristic equation

\[
r = \hat{r} + \tau \left( 1 - e^{-t/\tau} \right) u_o(\hat{r})
\]
Initial conditions - Single eyewall

\[ u_0(r)/u_{\text{max}} \]

\[ a \cdot u'_0(r)/u_{\text{max}} \]

\[ v_0(r)/v_{\text{max}} \]

\[ a \cdot \zeta_0(r)/(4 \cdot v_{\text{max}}) \]
Results - Single eyewall

![Diagram showing wind speed and direction over time and radius for a single eyewall.]
Results - Single eyewall
Results - Single eyewall

\[ \begin{align*}
\text{Radius (km)} & \\
0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 \\
\text{\( w \) (cm s}^{-1}) & \\
0 & 20 & 40 & 60 & 80 & 100 \\
\text{\( \zeta \) (10}^{-4} \text{ s}^{-1}) & \\
0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 \\
\text{\( t = 0.00 \text{ hr} \) & \( t = 1.69 \text{ hr} \) &}
\end{align*} \]
Time of shock formation

We can solve for the time of shock formation through

\[ \text{Time of shock formation} = t_s = -\tau \ln \left( 1 + \frac{1}{\tau u'_0(\hat{r}_s)} \right) \]

Radius of shock formation

\[ r_s = \hat{r}_s - \frac{u_0(\hat{r}_s)}{u'_0(\hat{r}_s)} \]
Conclusions

- 1-D solutions give a conceptual model of how diabatic heating influences intensity
- 1-D solutions inadequate at reproducing changes in modeled and observed storms
- 2-D solutions seem to have a more realistic structure and could be applied to modeled and observed storms
- 2-D solutions still neglect feedbacks from boundary layer
- Shock-like structures capture features in Hurricane Hugo (1989)
- Shocks control the size of the hurricane eye and the location of maximum Ekman pumping
Plans for creating a balanced model which includes frictional effects
Ooyama 1969a,b model design

Tropopause

Upper Troposphere

Lower Troposphere

Boundary Layer
Constant Depth

\( \rho_2 \)
\( h_2 \)
\( Q\psi \rightarrow \)
\( \psi_2 \)
\( v_2 \)

\( \rho_1 \)
\( h_1 \)
\( Q\psi \)
\( \psi_1 \)
\( v_1 \)

\( \rho_{0\psi}/\psi \rho_1 \)
\( h_0 \)
\( w \)

\( \tau_s \)

\( \psi_0 \)
\( v_0\psi/\psi v_1 \)
Ooyama 1969a,b differences

(a) \( v_0, v_1 \) and \( r \) vs. \( v_0 = v_1 \)

(b) \( w \) vs. \( r \)
Questions?