Diabatic and frictional forcing effects on the intensity and structure of tropical cyclones

Christopher J. Slocum

Colorado State University
Fort Collins, CO

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Motivation for theoretical work

"In order to make the best attainable forecast of the future weather, it would be desirable to express the physical laws as exactly as possible, and determine the initial conditions as precisely as possible. Yet the ultimate achievement of producing perfect forecasts, by applying equations already known to be exact to initial conditions already known to be precise, if such a feat were possible, would not by itself increase our understanding of the atmosphere, no matter how important it might be from other considerations."

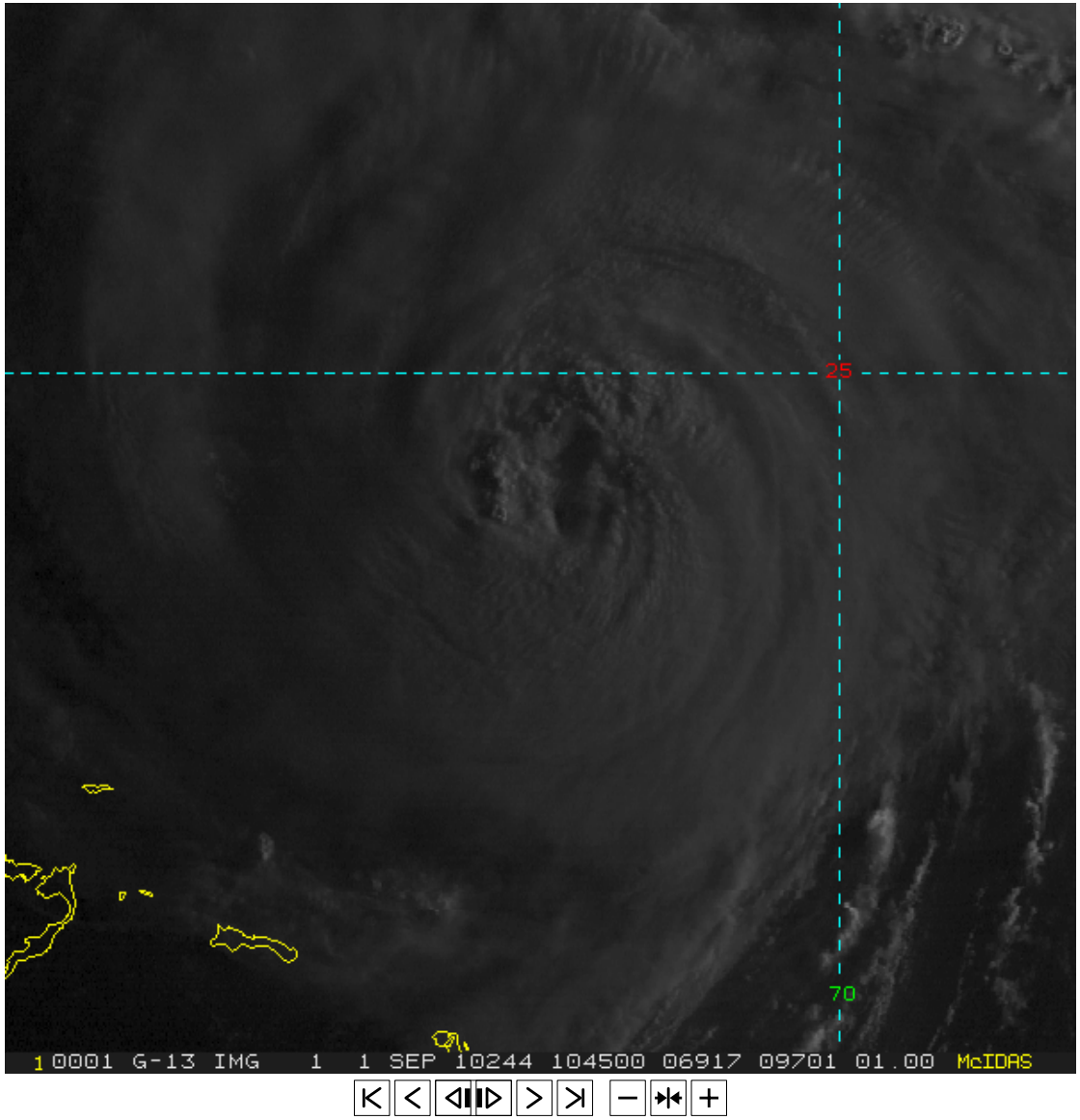
Lorenz, E. N., 1960: Maximum Simplification of the Dynamic Equations. Tellus, 12, 243–254.

Outline

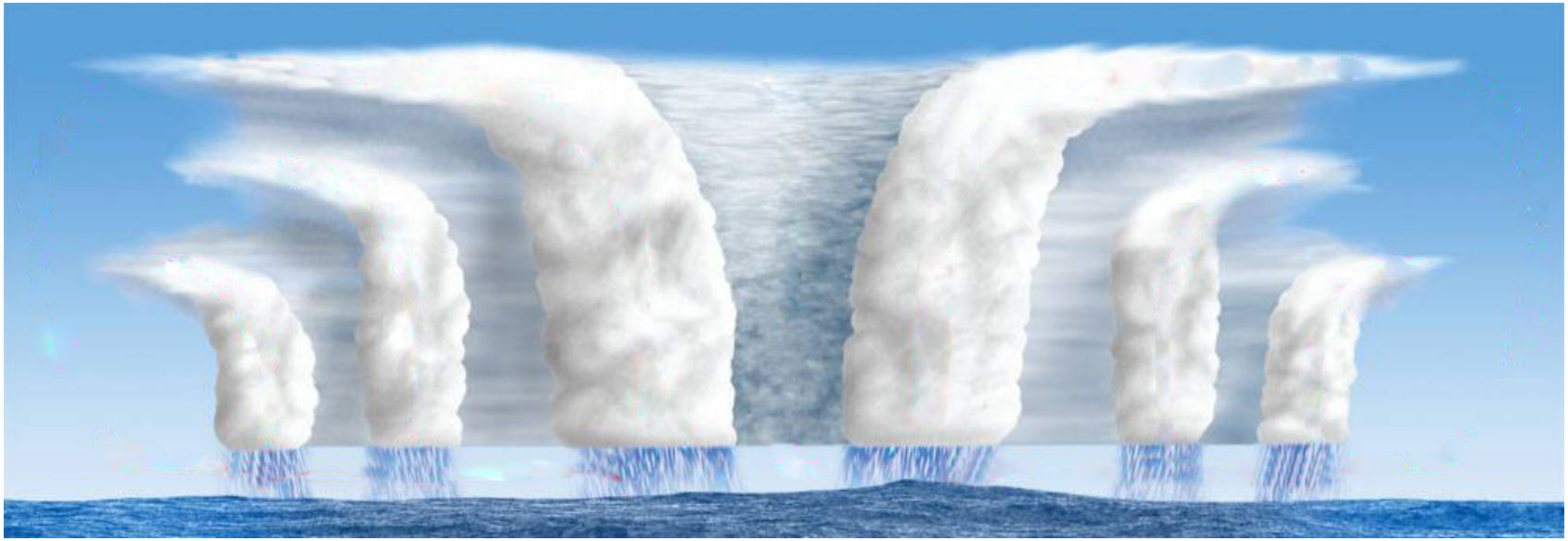
- Basic tropical cyclone structure and cylindrical coordinates
- The intensity forecasting problem
- Diabatic forcing with applications of Eliassen's balanced vortex model in the free atmosphere
- Shock-like structures in the tropical cyclone boundary layer due to frictional forcing
- Conclusions on the role of diabatic and frictional forcing
- Plans for creating a balanced model which includes frictional effects

Basic tropical cyclone structure and cylindrical coordinates

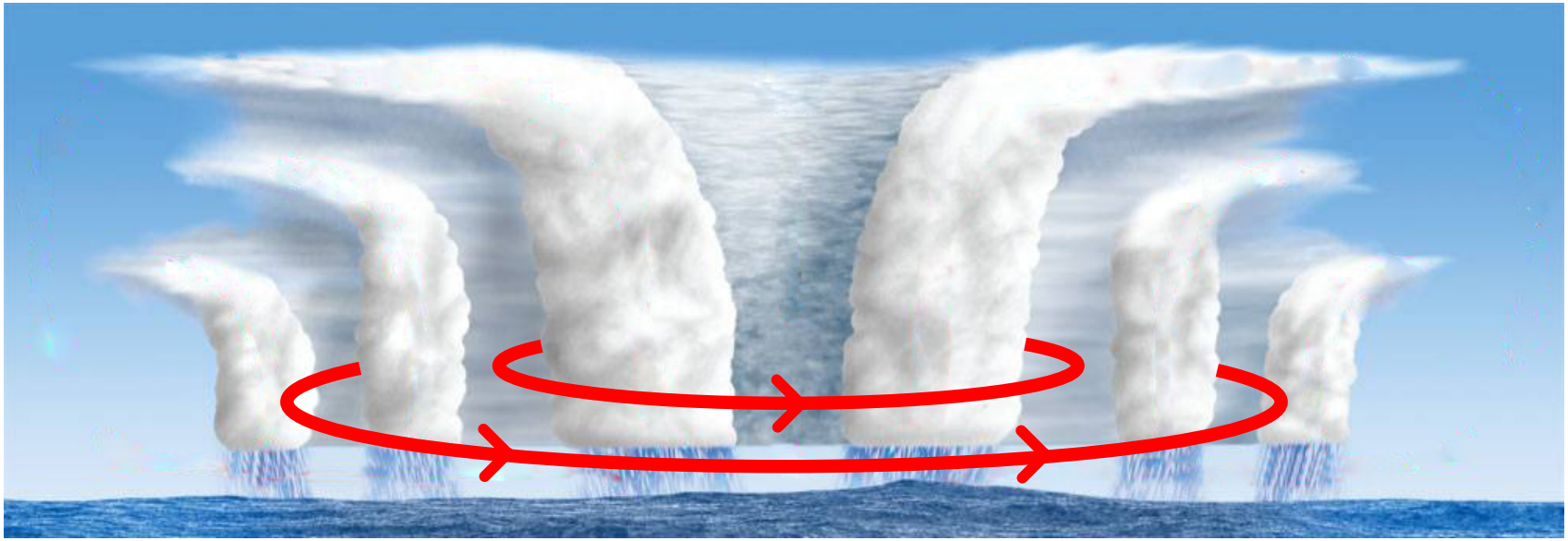
Visible imagery for Hurricane Earl on 1 Sept 2010



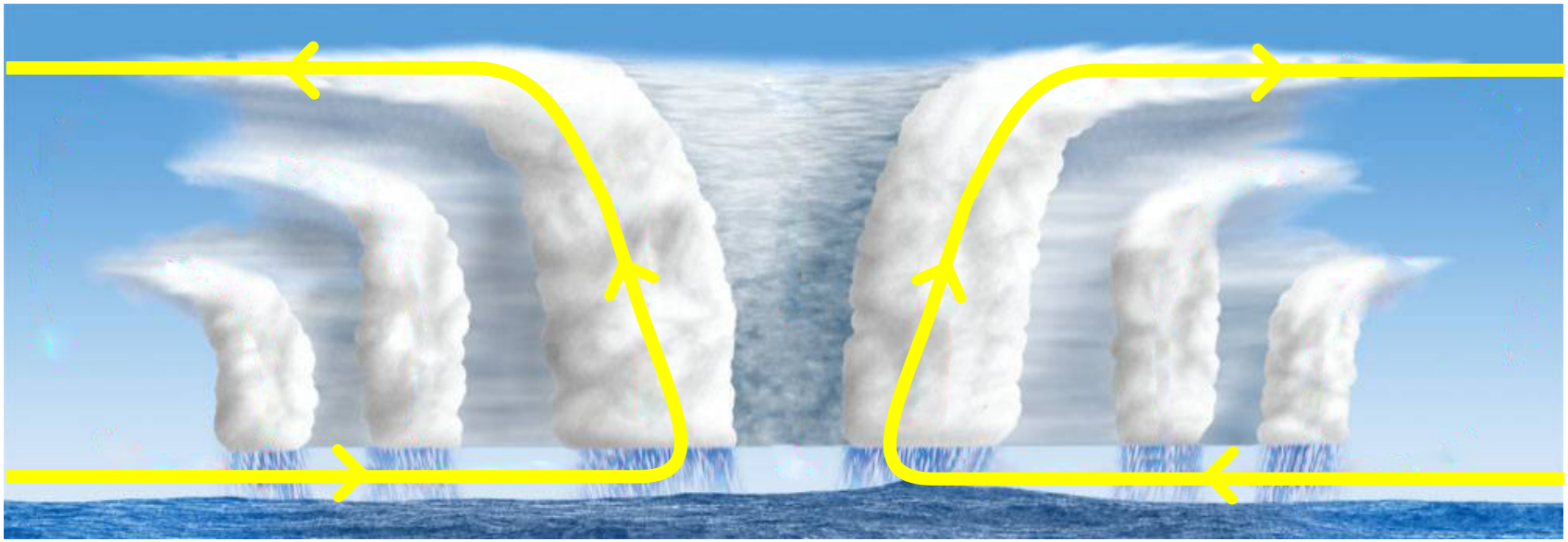
Underneath the clouds



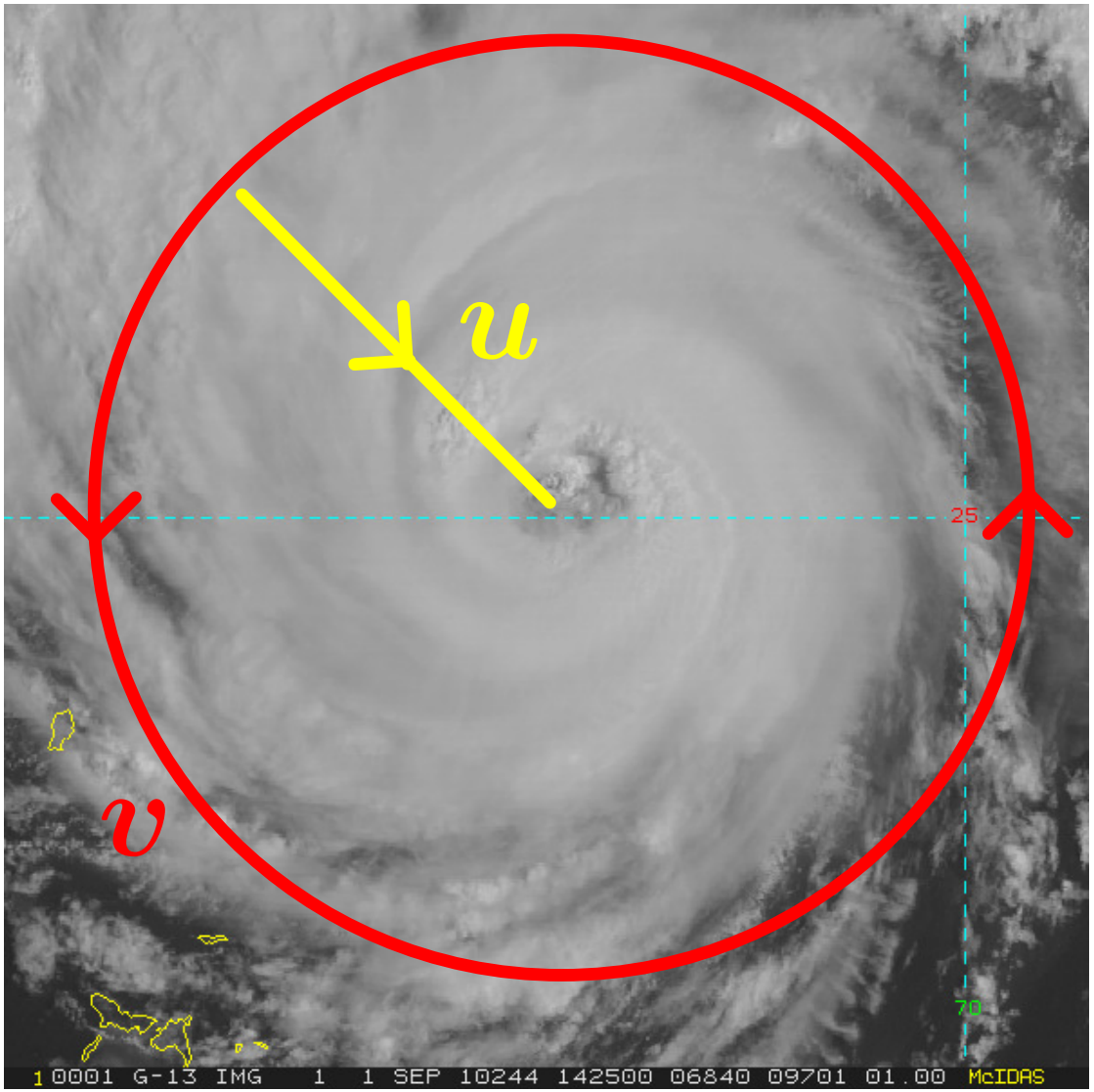
Underneath the clouds



Underneath the clouds



Cylindrical Coordinates



Vorticity

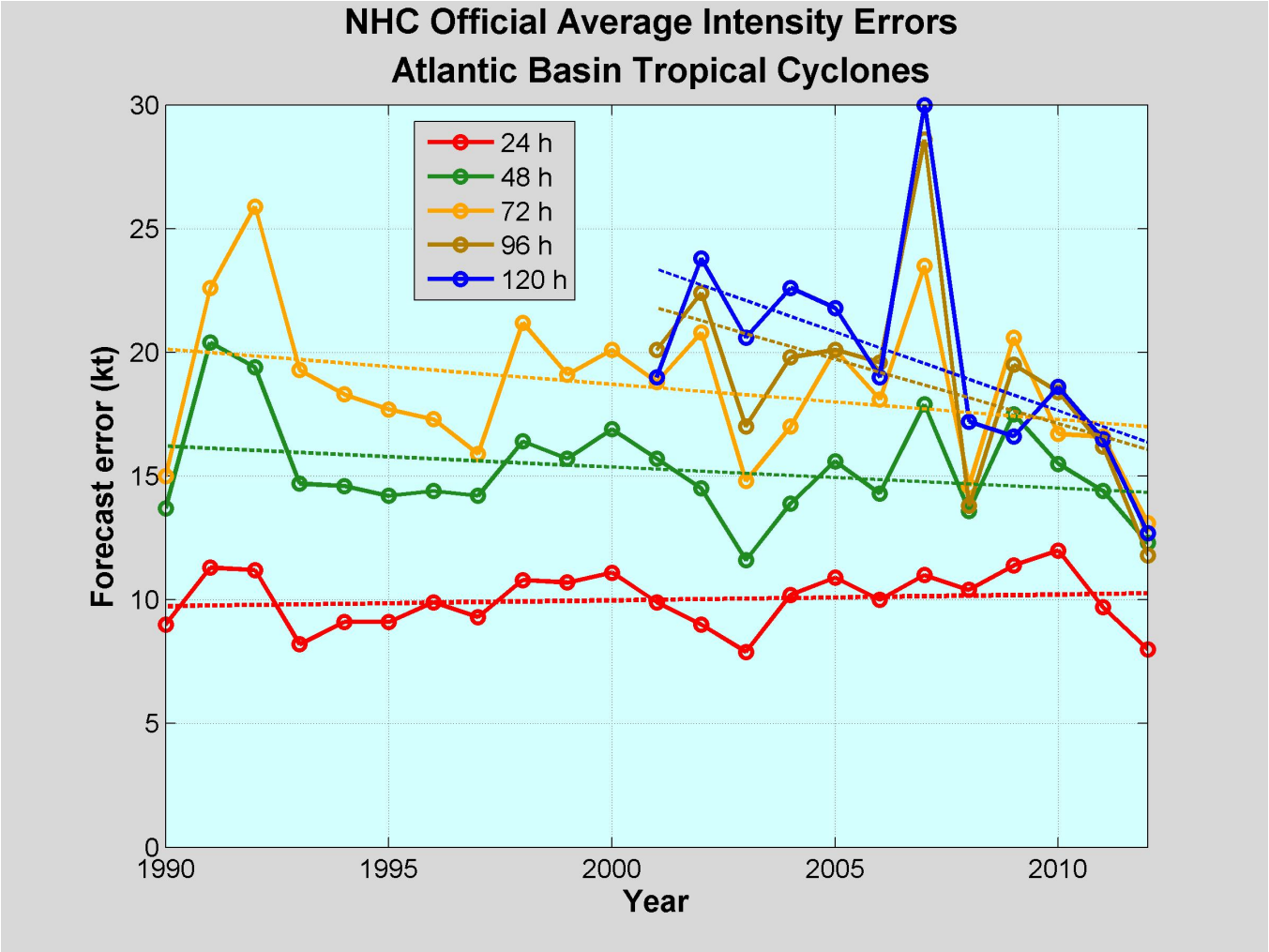
$$\zeta = \frac{\partial(rv)}{r\partial r}$$

Divergence

$$\delta = \frac{\partial(ru)}{r\partial r}$$

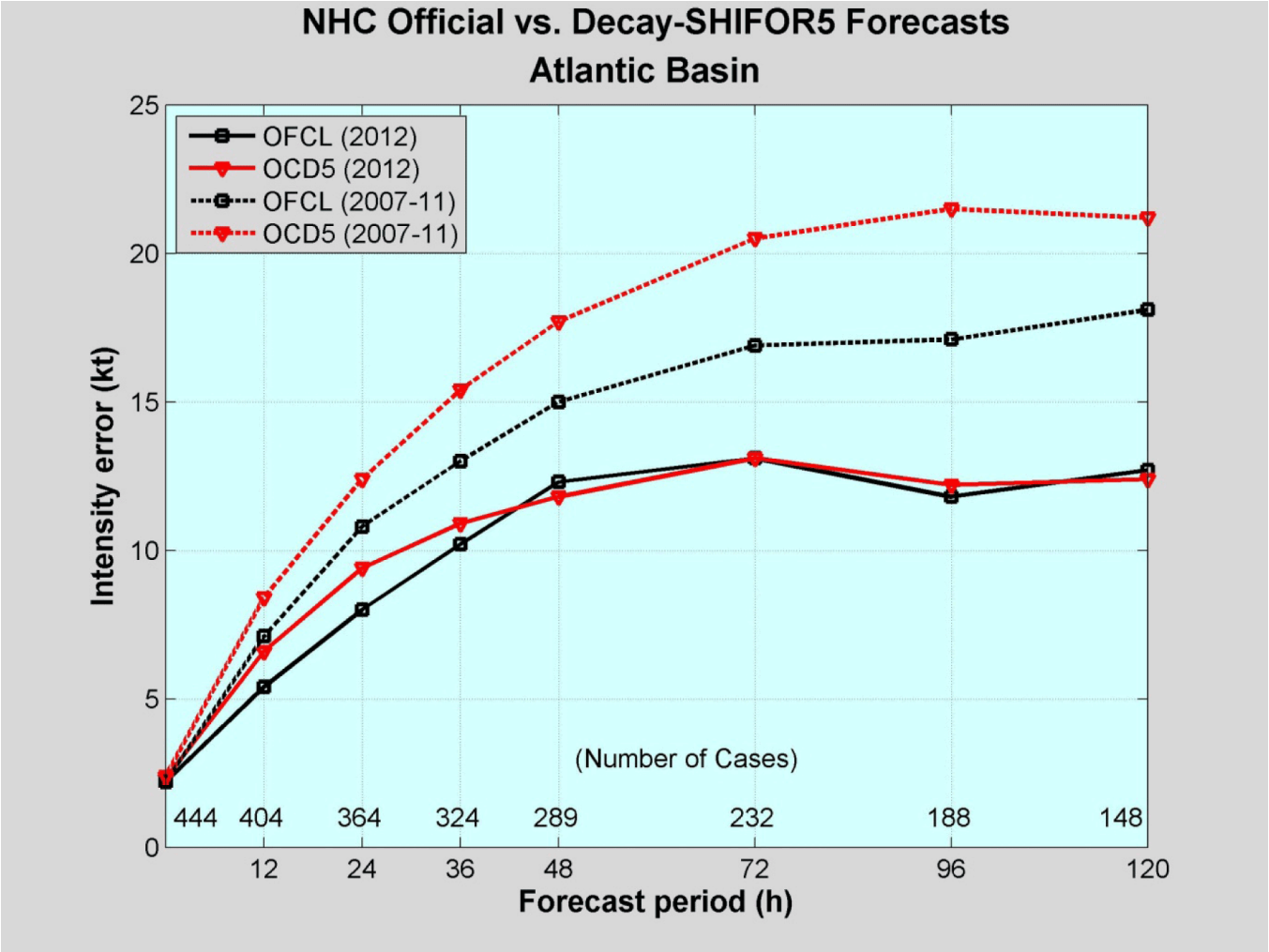
The intensity forecasting problem

Atlantic Intensity Error Trends (1990-2012)



From Cangialosi and Franklin (2013)

Atlantic Intensity Error for 2012 and 2007-2011



From Cangialosi and Franklin (2013)

Diabatic forcing with applications of Eliassen's balanced vortex model in the free atmosphere

Why Evaluate Balanced Vortex Model Applications?

- Eliassen's Balanced Vortex Model potentially offers a
  - simple
  - fast
  - elegant

  way to explain tropical cyclone intensity change through diabatic heating.

Assumptions for the Balanced Vortex Model

- Inviscid
- Axisymmetric
- Quasistatic
- Gradient Balanced
- Stratified
- f-plane

Balanced Vortex Model – Governing Equations

Gradient Balance: $$\left(f + \frac{v}{r}\right) v = \frac{\partial \phi}{\partial r} \tag{1}$$

Tangential Velocity: $$\frac{\partial v}{\partial t} + u\left(f + \frac{\partial (rv)}{r \partial r}\right) + w\frac{\partial v}{\partial z} = 0 \tag{2}$$

Hydrostatic: $$\frac{\partial \phi}{\partial z} = \frac{g}{T_0} T \tag{3}$$

Continuity: $$\frac{\partial (ru)}{r \partial r} + \frac{\partial (\rho w)}{\rho \partial z} = 0 \tag{4}$$

Thermodynamic: $$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\left(\frac{\partial T}{\partial z} + \frac{RT}{c_p H}\right) = \frac{Q}{c_p} \tag{5}$$

Can solve for 1 of 2 PDEs

Transverse circulation

$$\frac{\partial}{\partial r}\left(A\frac{\partial(r\psi)}{r\partial r}+B\frac{\partial\psi}{\partial z}\right)+\frac{\partial}{\partial z}\left(B\frac{\partial(r\psi)}{r\partial r}+C\frac{\partial\psi}{\partial z}\right)=\frac{g}{c_pT_\mathrm{o}}\frac{\partial Q}{\partial r}$$

Geopotential Tendency

$$\frac{\partial}{r\partial r}\left(r\frac{A}{D}\frac{\partial\phi_t}{\partial r}+r\frac{B}{D}\frac{\partial\phi_t}{\partial z}\right)+\frac{\partial}{\partial z}\left(\frac{B}{D}\frac{\partial\phi_t}{\partial r}+\frac{C}{D}\frac{\partial\phi_t}{\partial z}\right)=$$
$$\frac{g}{c_pT_\mathrm{o}}\left[\frac{\partial}{r\partial r}\left(r\frac{B}{D}Q\right)+\frac{\partial}{\partial z}\left(\frac{C}{D}Q\right)\right]$$

Can solve for 1 of 2 PDEs

Transverse circulation

$$\frac{\partial}{\partial r}\left(A\frac{\partial(r\psi)}{r\partial r}+B\frac{\partial\psi}{\partial z}\right)+\frac{\partial}{\partial z}\left(B\frac{\partial(r\psi)}{r\partial r}+C\frac{\partial\psi}{\partial z}\right)=\frac{g}{c_pT_\mathrm{o}}\frac{\partial Q}{\partial r}$$

Geopotential Tendency

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$$\frac{g}{c_pT_\mathrm{o}}\left[\frac{\partial}{r\partial r}\left(r\frac{B}{D}Q\right)+\frac{\partial}{\partial z}\left(\frac{C}{D}Q\right)\right]$$

Static Stability

$$\rho A = \frac{g}{T_0}\left(\frac{\partial T}{\partial z} + \frac{R_d T}{c_p H}\right)$$

Baroclinicity

$$\rho B = -\frac{g}{T_0}\frac{\partial T}{\partial r} = -\left(f + \frac{2v}{r}\right)\frac{\partial v}{\partial z}$$

Inertial Stability

$$\rho C = \left(f + \frac{2v}{r}\right)\left(f + \frac{\partial (rv)}{r\partial r}\right)$$

Elliptic Condition

$$D = AC - B^2 > 0$$

1-D Simplified Geopotential Tendency

2-D equation with zero Baroclinicity and constant static stability

$$N^2 \frac{\partial}{r\partial r}\left(\frac{r}{\hat{f}^2}\frac{\partial \phi_t}{\partial r}\right) + \left(\frac{\partial}{\partial z} - \frac{1}{H}\right)\frac{\partial \phi_t}{\partial z} = \frac{g}{c_p T_o}\left(\frac{\partial}{\partial z} - \frac{1}{H}\right) Q$$

1-D ordinary differential equation (radial structure only)

$$\hat{T}_t - \frac{d}{rdr}\left(\ell^2 r \frac{d\hat{T}_t}{dr}\right) = \frac{\hat{Q}}{c_p}$$

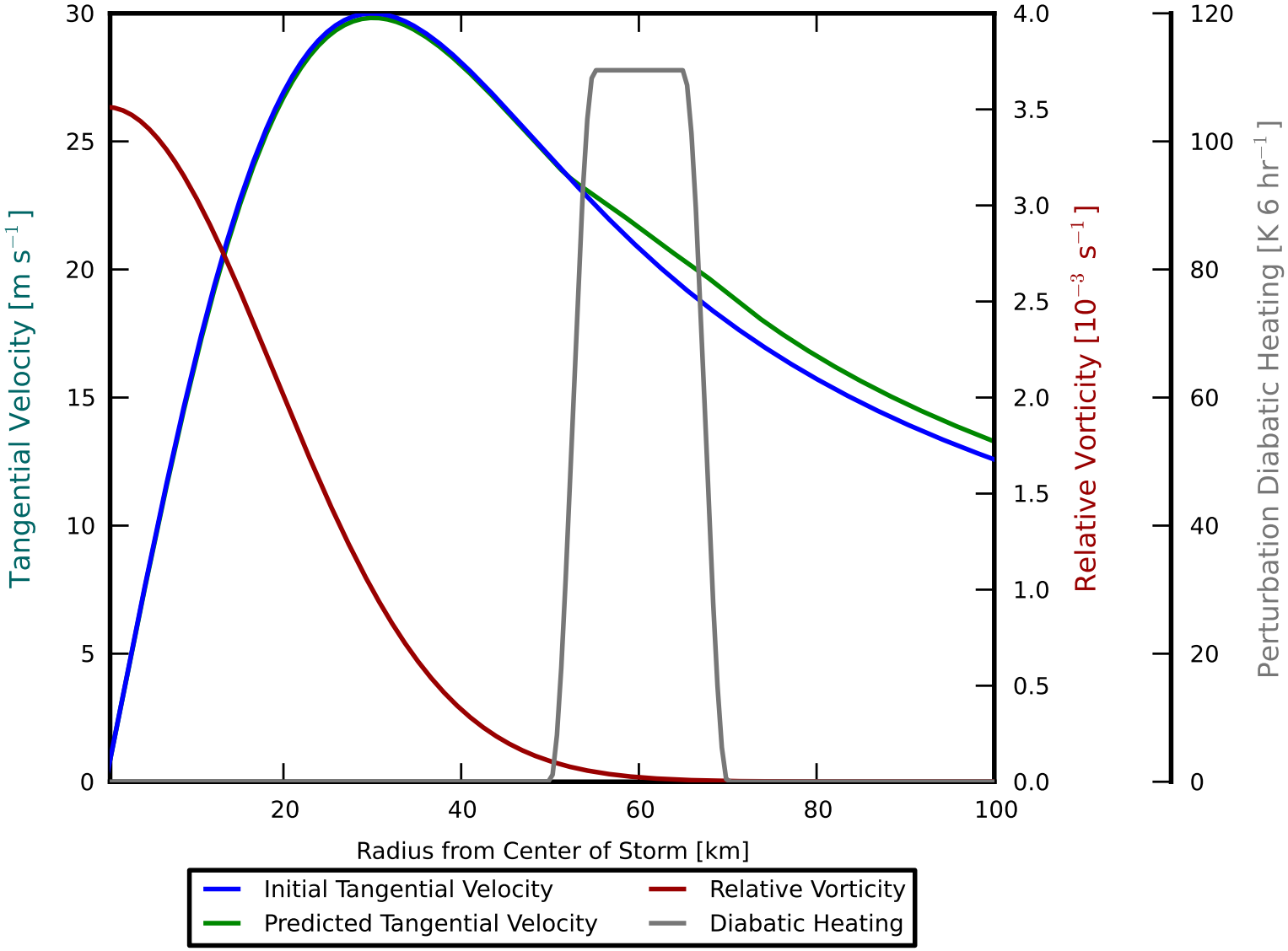
$$\frac{d\hat{T}_t}{dr} = 0 \quad \text{at } r = 0$$

$$\frac{d\hat{T}_t}{dr} = -\left(\frac{K_1(b/\ell_o)}{\ell_o K_o(b/\ell_o)}\right)\hat{T}_t \quad \text{at } r = b$$

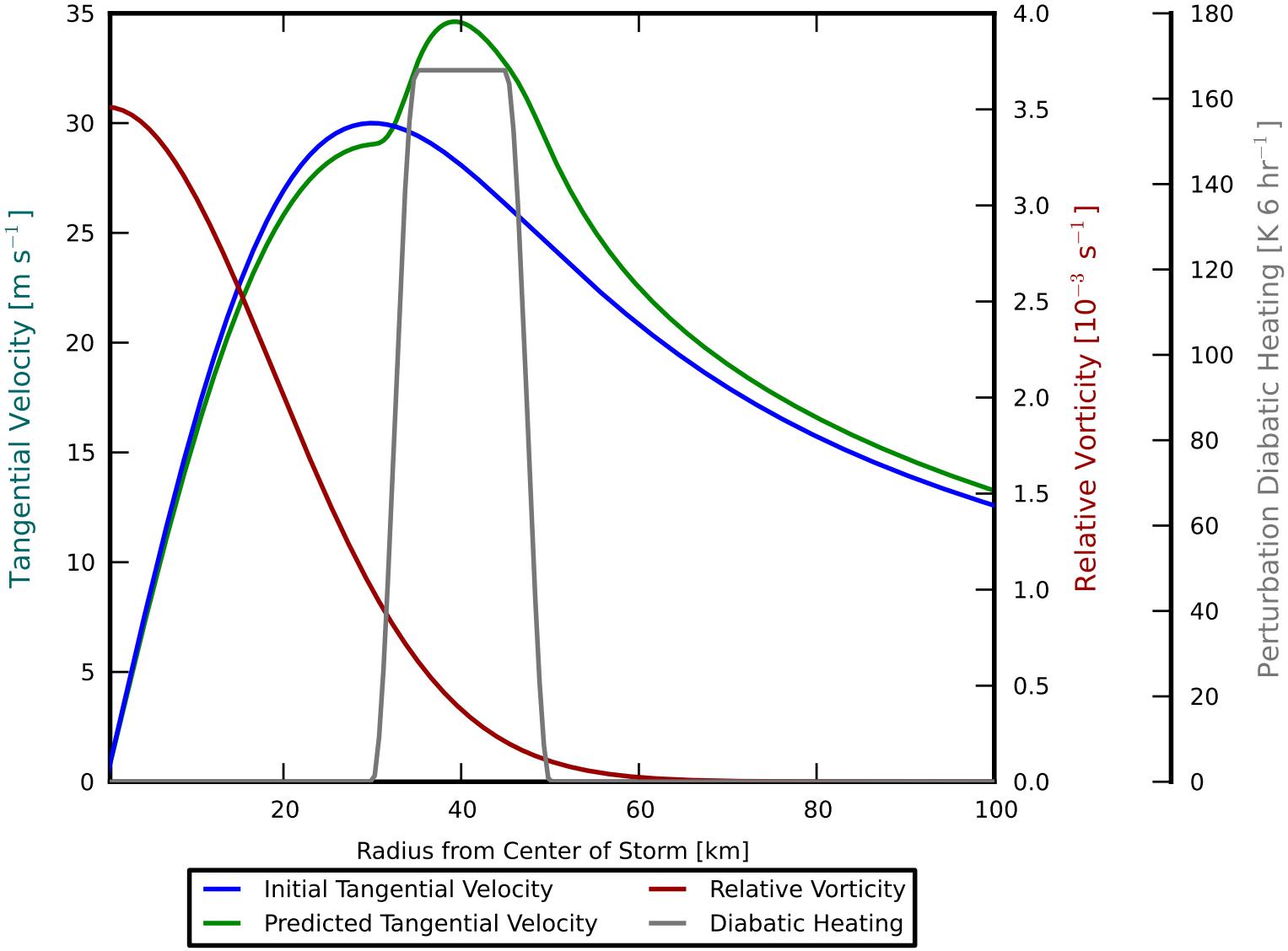
$(\hat{\ })$ represents the radial part

Based on Musgrave et al. 2012

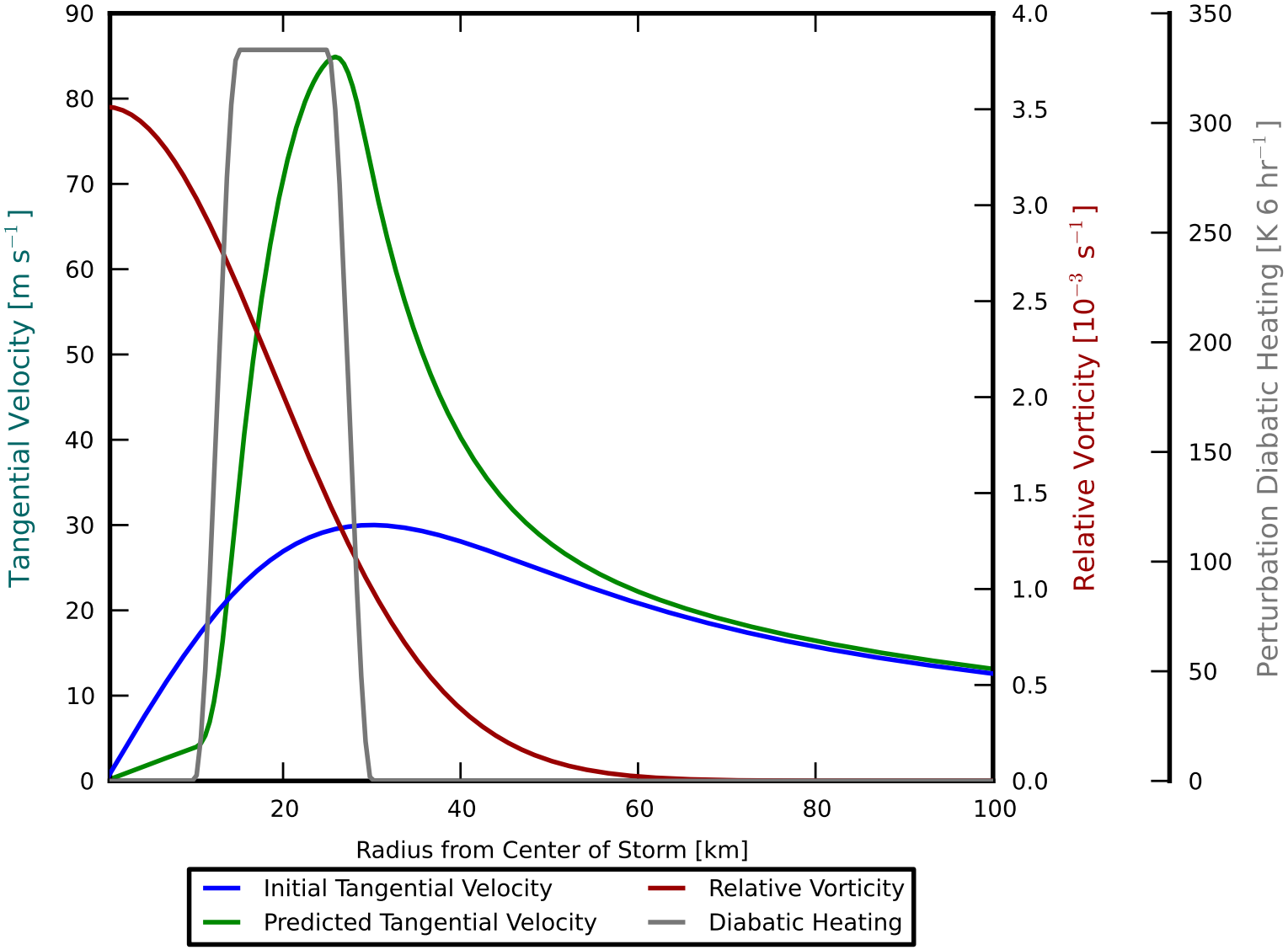
Idealized 1-D Balanced Solutions



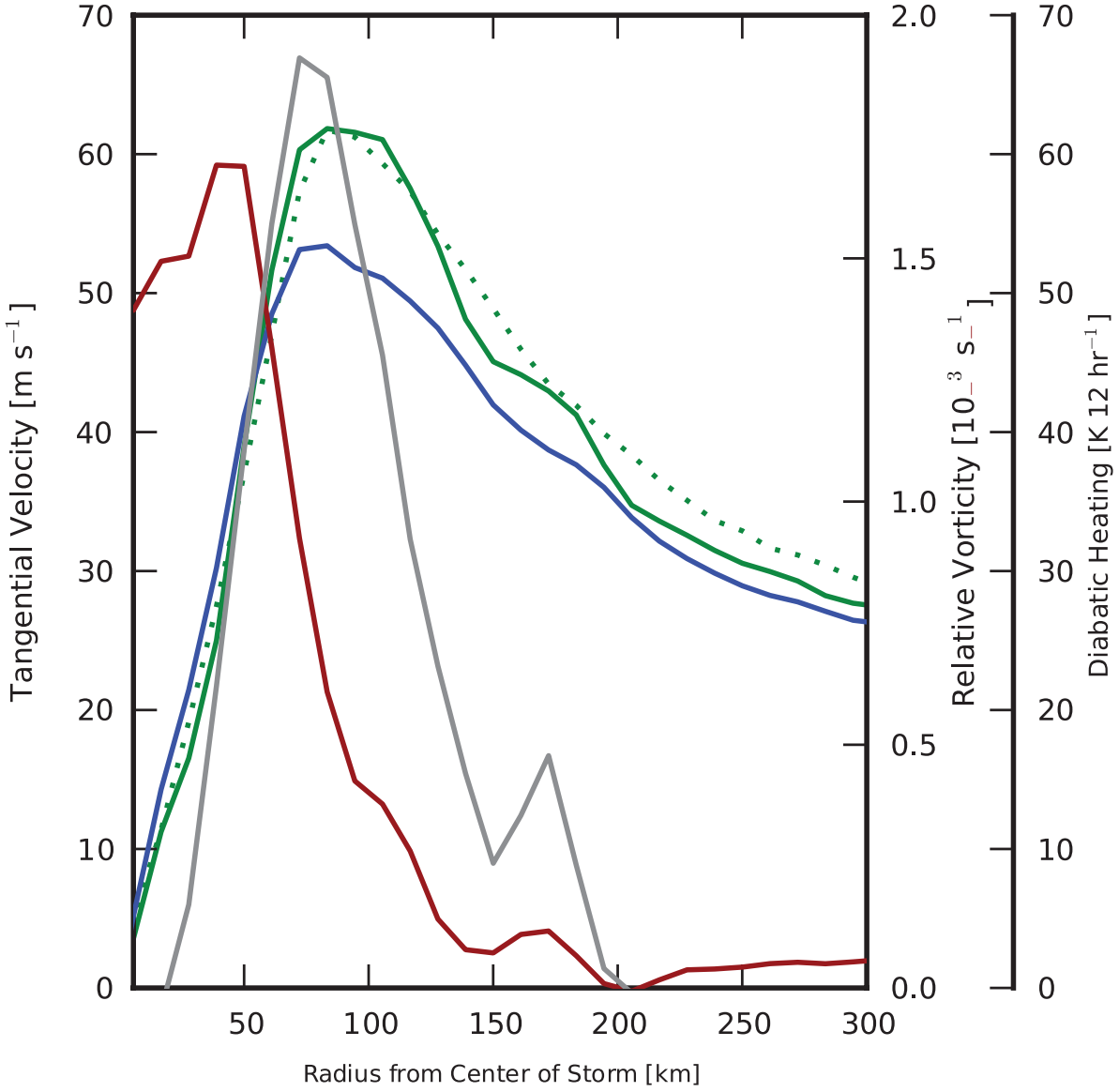
Idealized 1-D Balanced Solutions



Idealized 1-D Balanced Solutions



Irene – 78 hr HWRF forecast on 18 UTC 21 Aug 2011



Lessons from 1-D Geopotential solutions

- Defines a powerful conceptual model for velocity tendency from
  - Diabatic heating
  - Radius of maximum wind
  - High inertial stability region

- In models and observed storms
  - Lacks vertical structure
  - Cannot dissipate storms
  - Resolution of diabatic heating too low

Can solve for 1 of 2 PDEs

Transverse circulation

$$\frac{\partial}{\partial r}\left(A\frac{\partial(r\psi)}{r\partial r}+B\frac{\partial\psi}{\partial z}\right)+\frac{\partial}{\partial z}\left(B\frac{\partial(r\psi)}{r\partial r}+C\frac{\partial\psi}{\partial z}\right)=\frac{g}{c_pT_\mathrm{o}}\frac{\partial Q}{\partial r}$$

Geopotential Tendency

$$\frac{\partial}{r\partial r}\left(r\frac{A}{D}\frac{\partial\phi_t}{\partial r}+r\frac{B}{D}\frac{\partial\phi_t}{\partial z}\right)+\frac{\partial}{\partial z}\left(\frac{B}{D}\frac{\partial\phi_t}{\partial r}+\frac{C}{D}\frac{\partial\phi_t}{\partial z}\right)=$$
$$\frac{g}{c_pT_\mathrm{o}}\left[\frac{\partial}{r\partial r}\left(r\frac{B}{D}Q\right)+\frac{\partial}{\partial z}\left(\frac{C}{D}Q\right)\right]$$

2-D Balanced Solutions

Return to the transverse circulation

$$\frac{\partial}{\partial r}\left(A\frac{\partial(r\psi)}{r\partial r}+B\frac{\partial\psi}{\partial z}\right)+\frac{\partial}{\partial z}\left(B\frac{\partial(r\psi)}{r\partial r}+C\frac{\partial\psi}{\partial z}\right)=\frac{g}{c_pT_\mathrm{o}}\frac{\partial Q}{\partial r}$$

Boundary conditions

$$\psi(r,\mathrm{o})=\psi_\mathrm{o}(r),\quad \psi(r,z_T)=0,$$

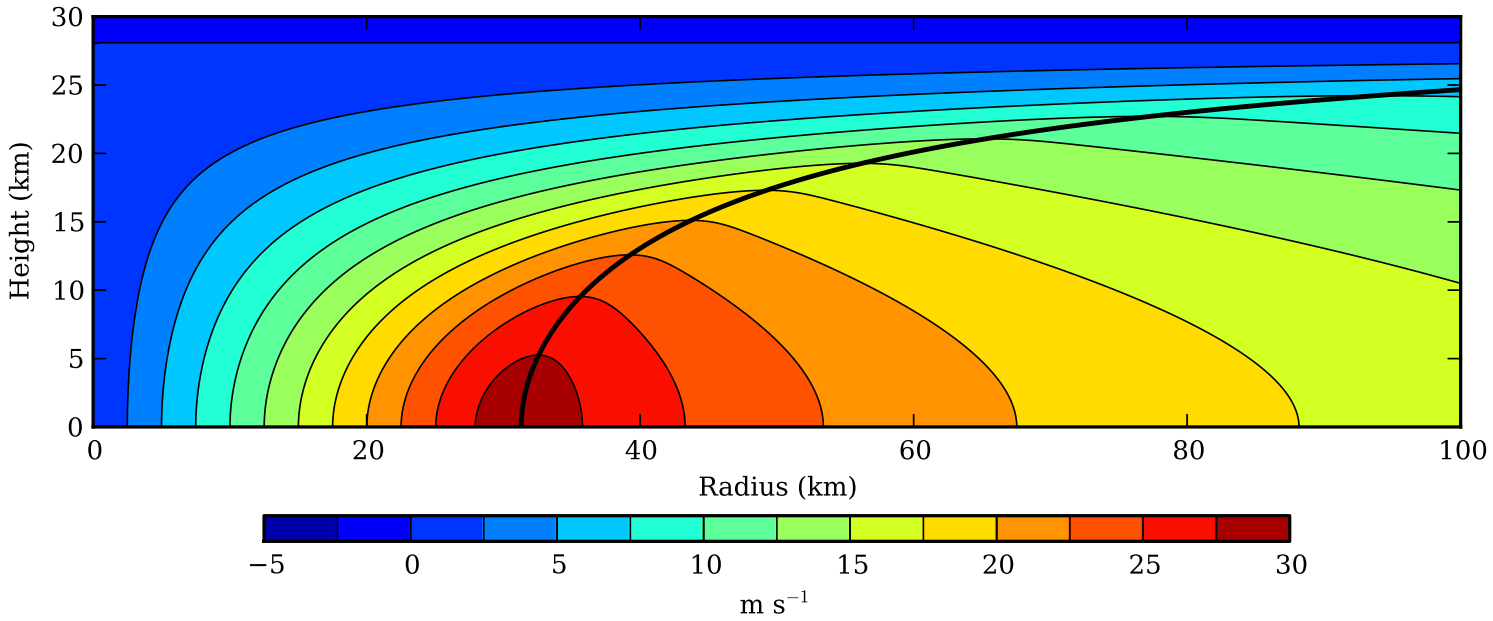
$$\psi(\mathrm{o},z)=0,\quad \text{and}$$

$$\frac{\partial\psi}{\partial r}=-\frac{\psi}{\ell}\quad \text{at}\quad r=r_B$$

Solve the transverse circulation via a successive over-relaxation iterative method

Include baroclinicity and non-constant static stability

Tangential velocity

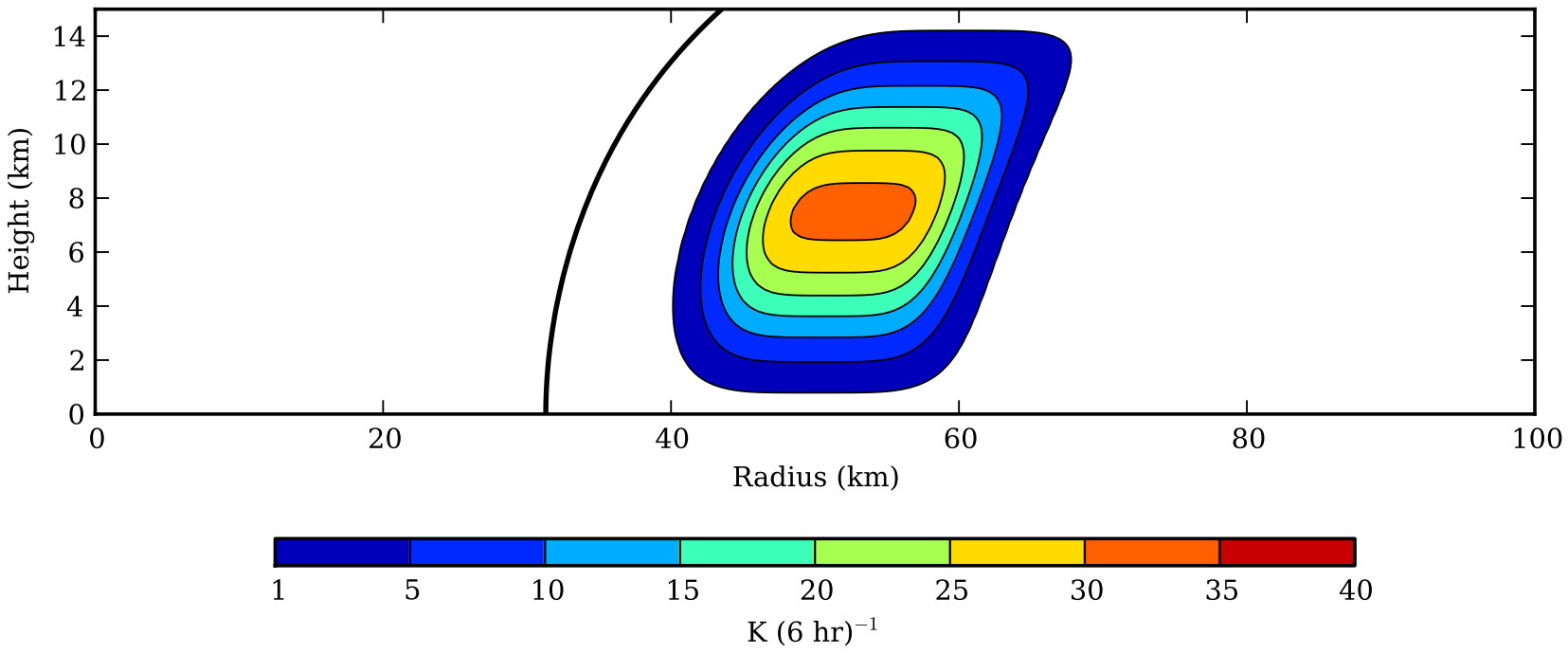


Modified Rankine vortex with smoothing

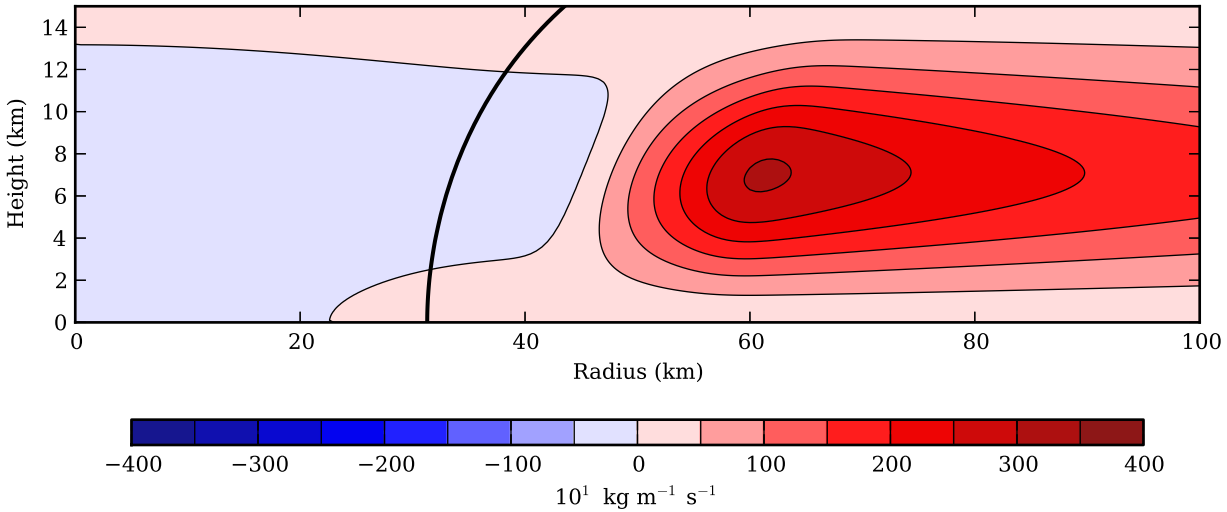
$$v(r,z) = \frac{1}{2}\zeta_o(z)\begin{cases} r & 0 \leq r \leq r_m(z) \\ r_m^{\alpha+1}(z)/r^{\alpha} & r_m(z) \leq r < \infty \end{cases}$$

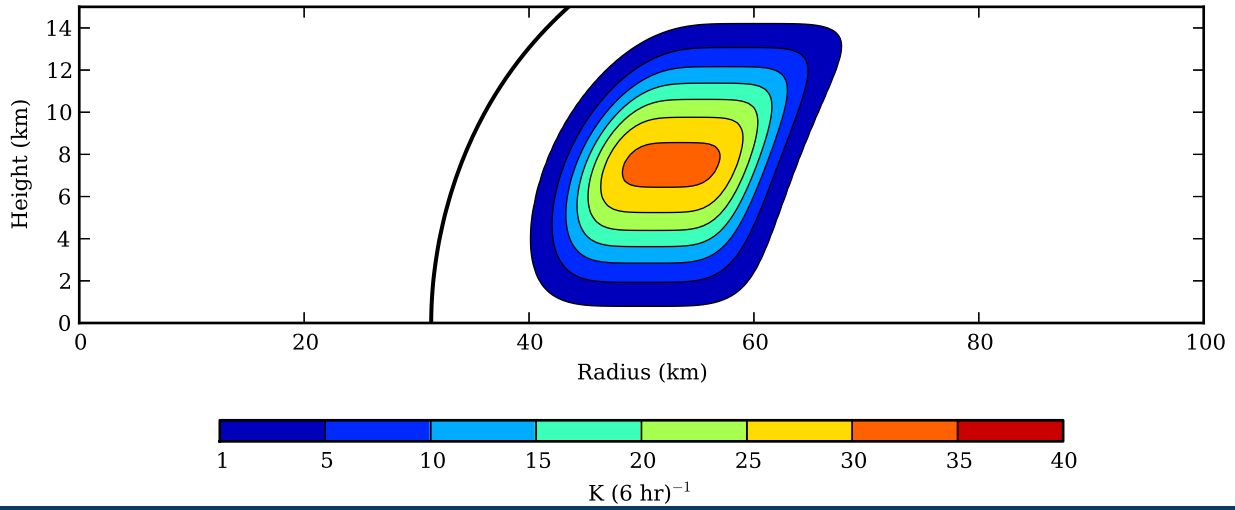
$$r_m(z) = \left(\frac{f}{f+\zeta_o(z)}\right)^{1/2} R_o$$

Diabatic heating

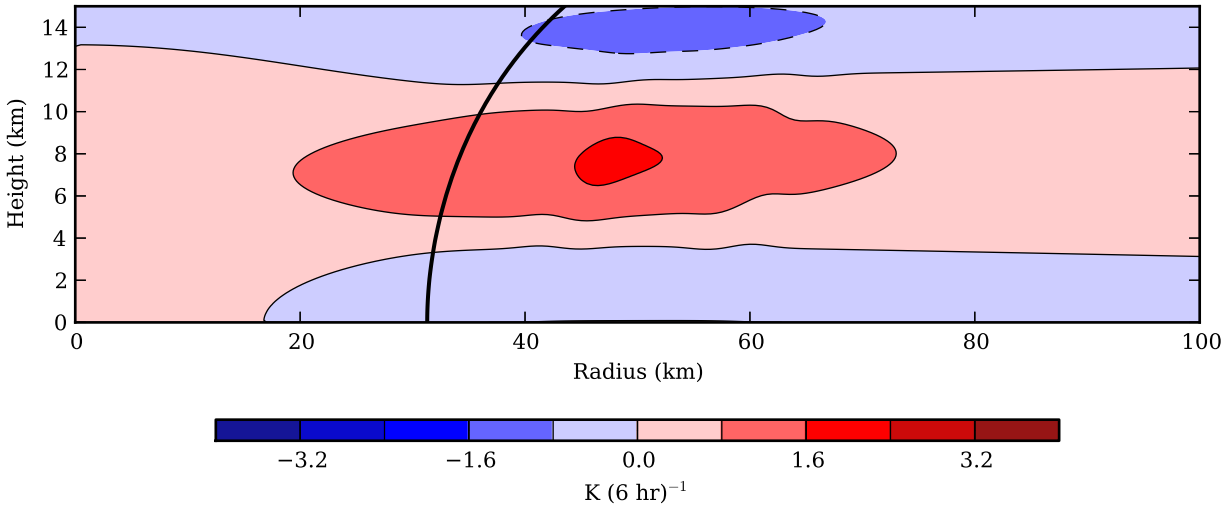


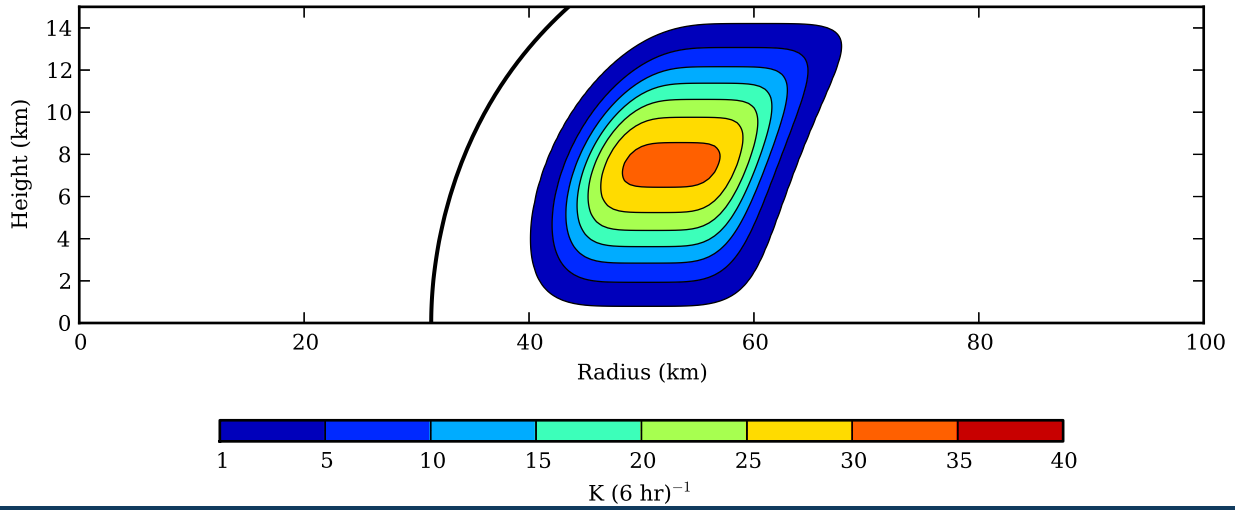
Results - Streamfunction



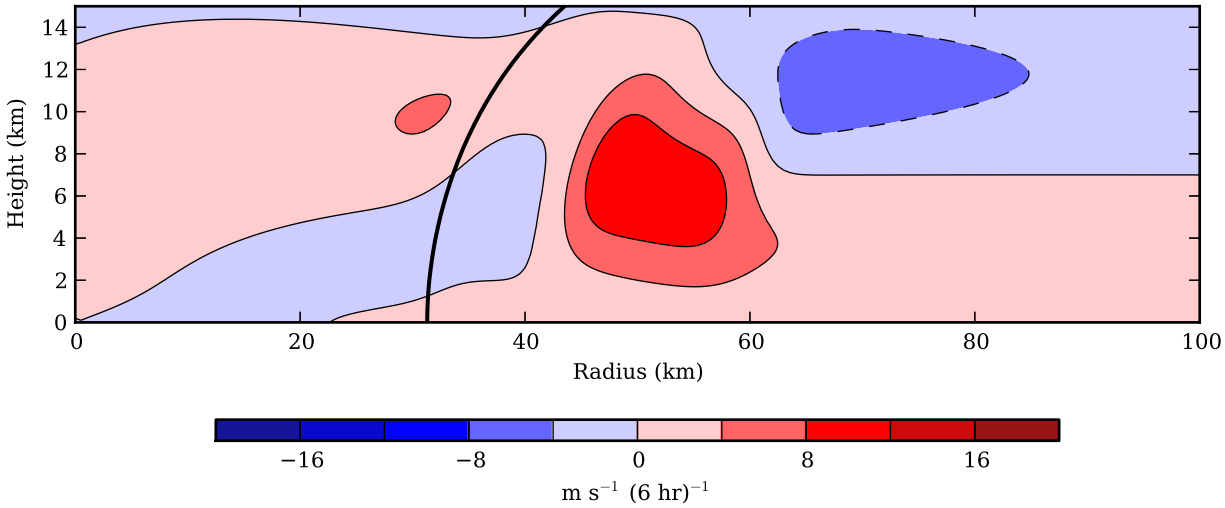


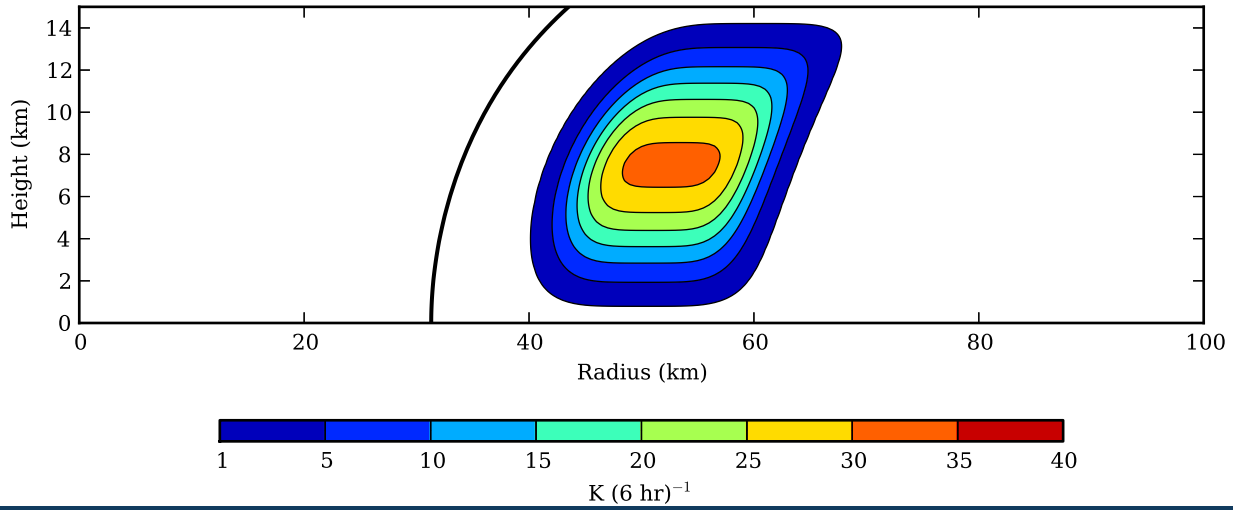
Results - Temperature Tendency





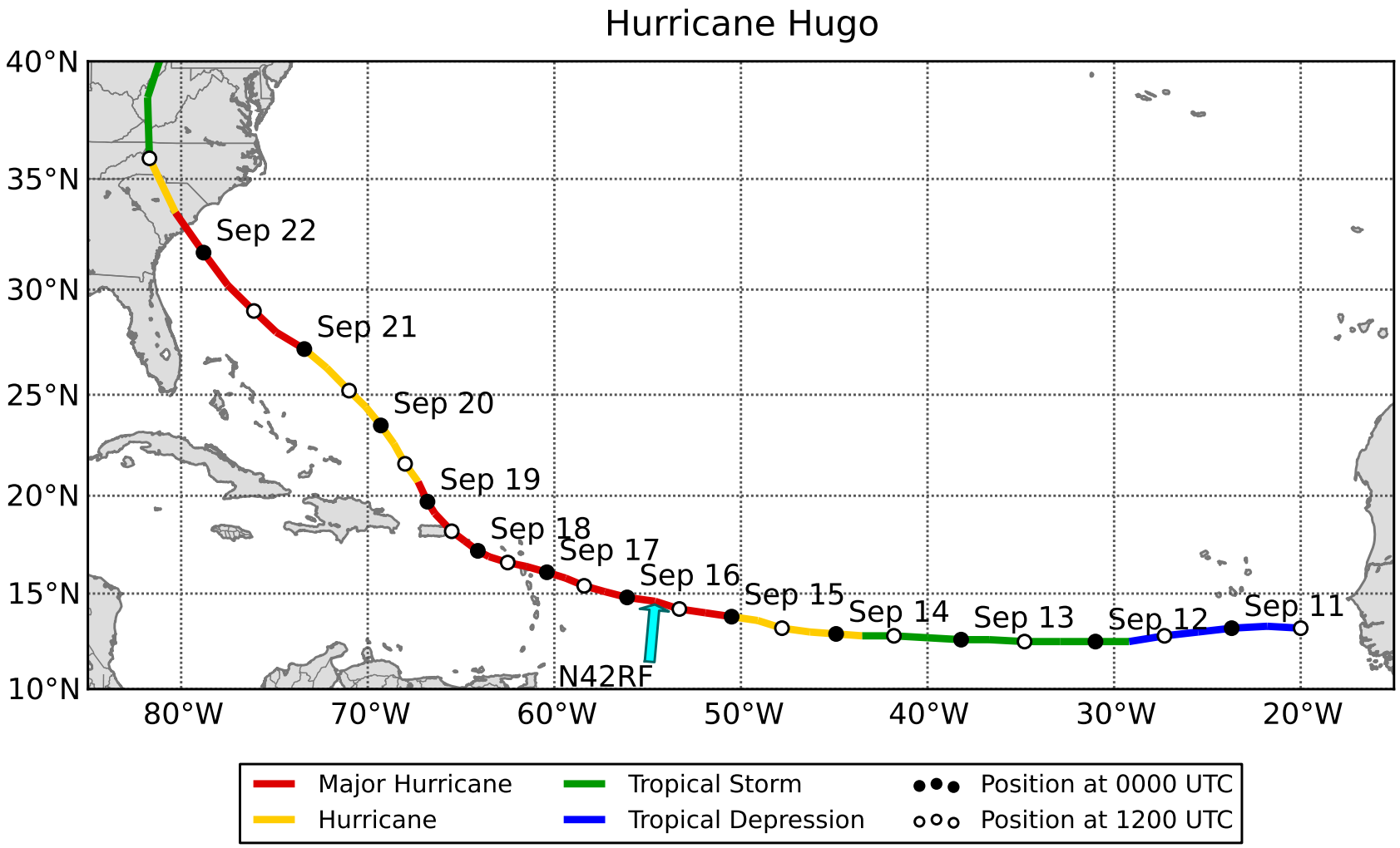
Results - Tangential Velocity Tendency



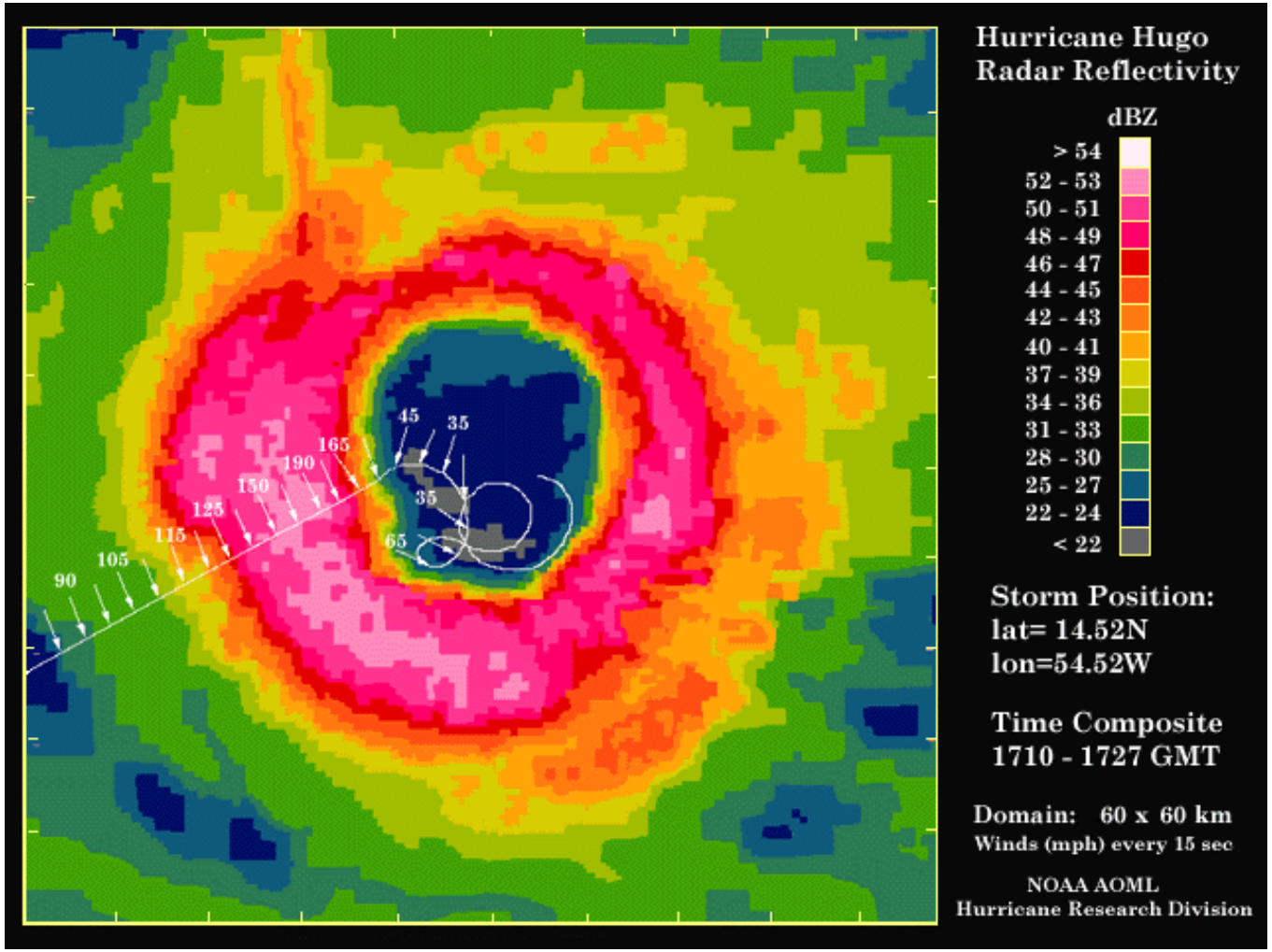


Shock-like structures in the tropical cyclone boundary layer due to frictional forcing

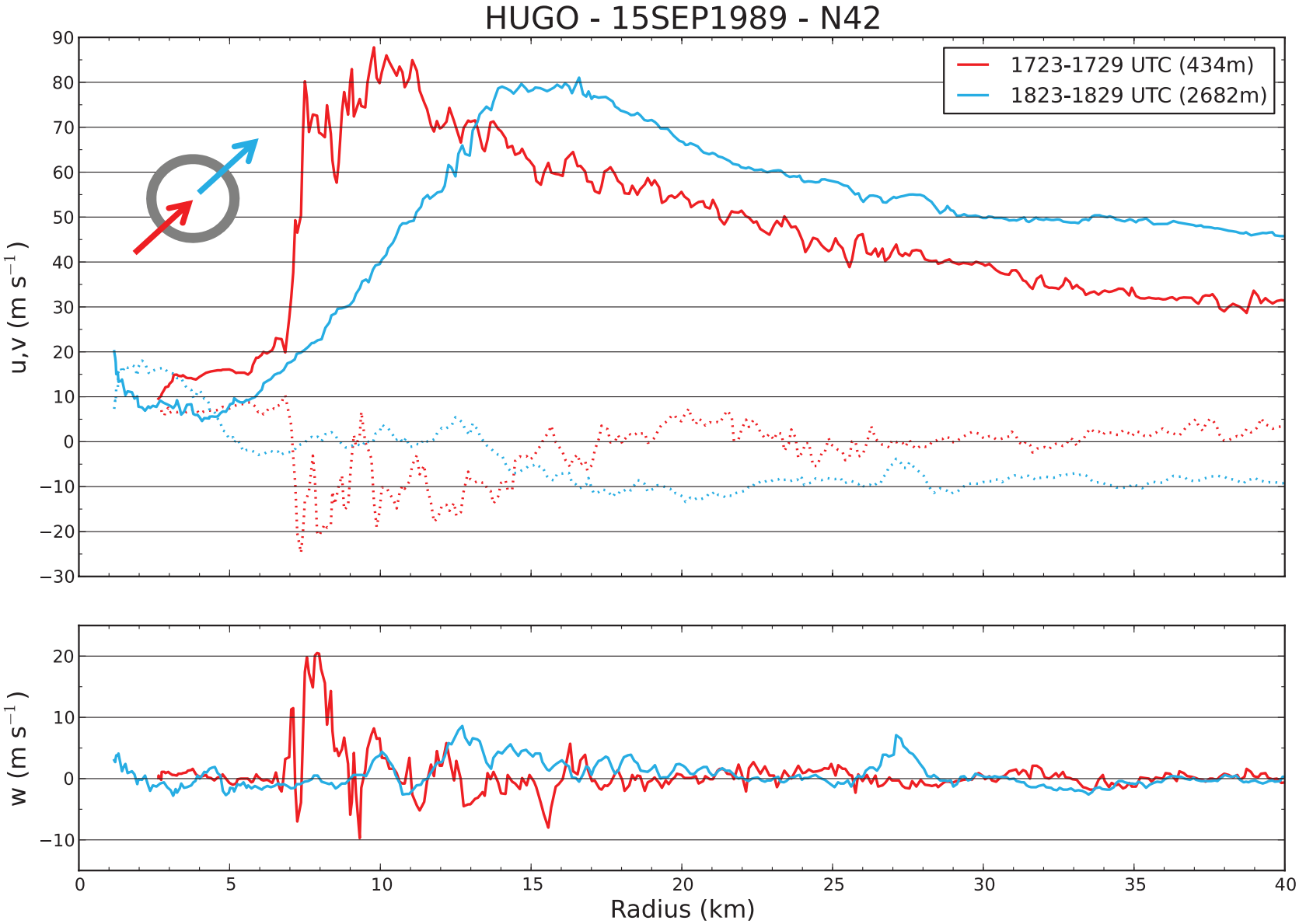
Hurricane Hugo (1989) – Track



Hurricane Hugo (1989) – Radar

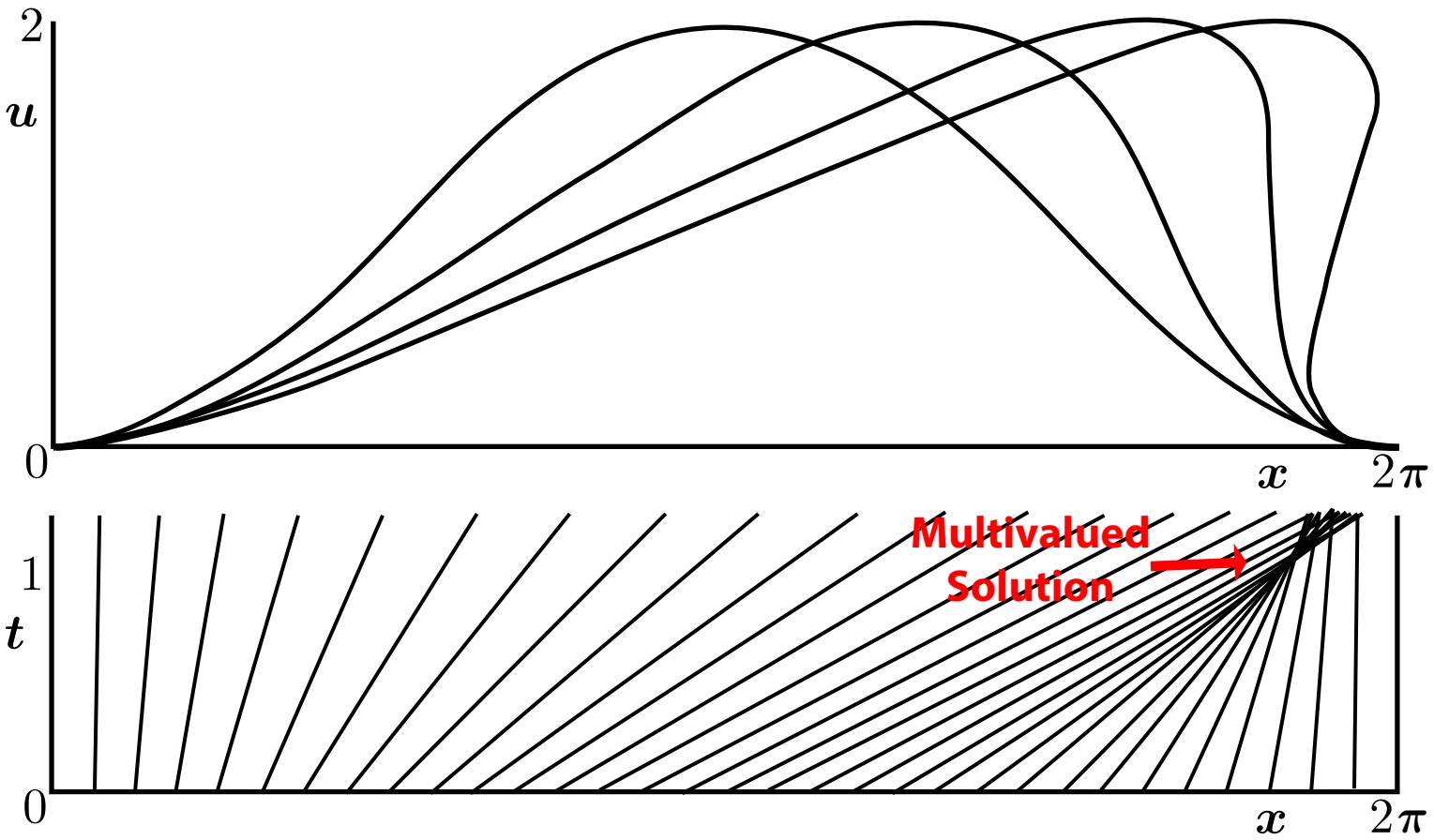


Hurricane Hugo (1989) – Radial Profile



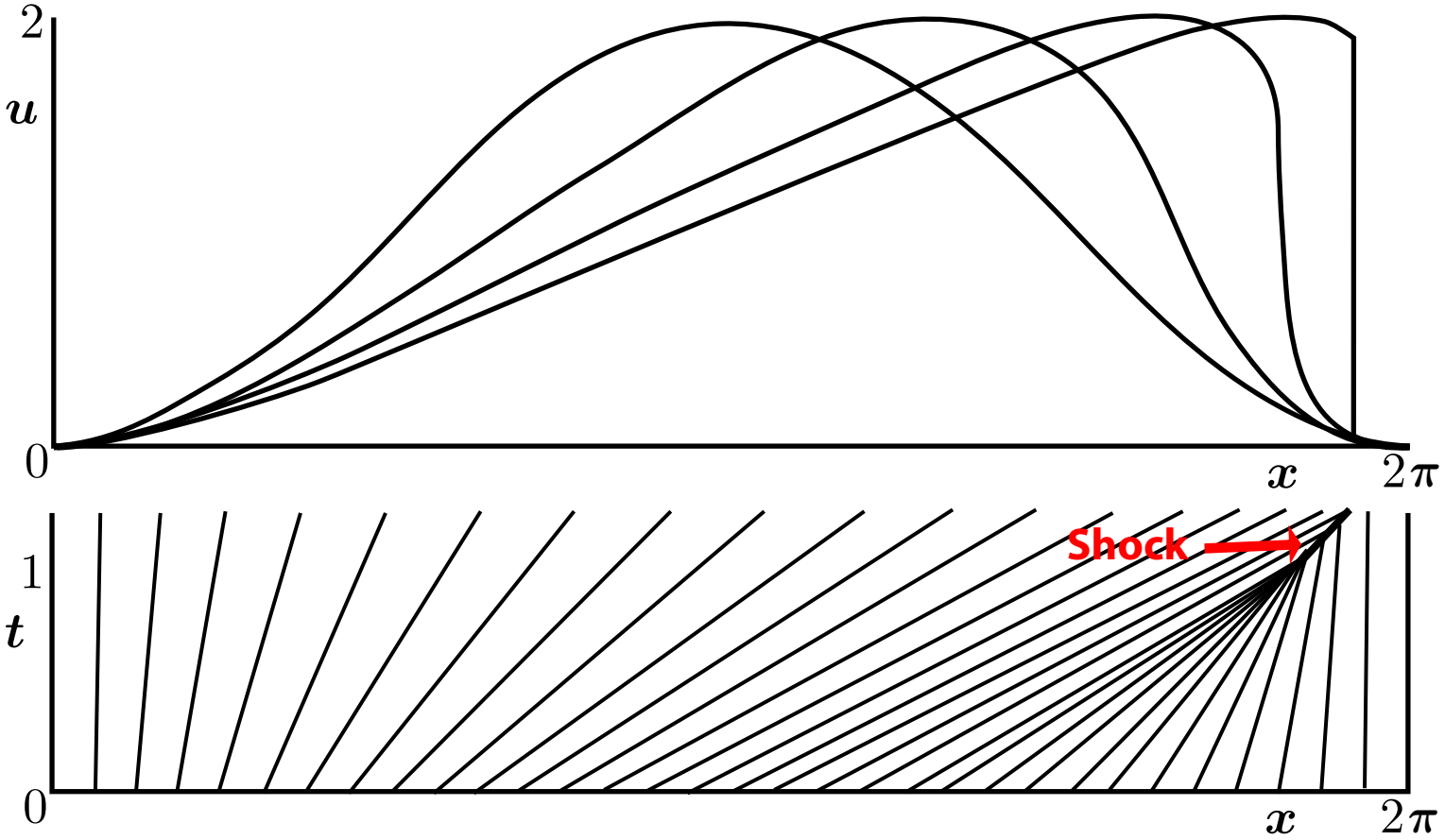
Inviscid Burgers' Equation

- Model for nonlinear wave propagation: $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$
- Example initial condition: $u(x, 0) = 1 - \cos(x)$

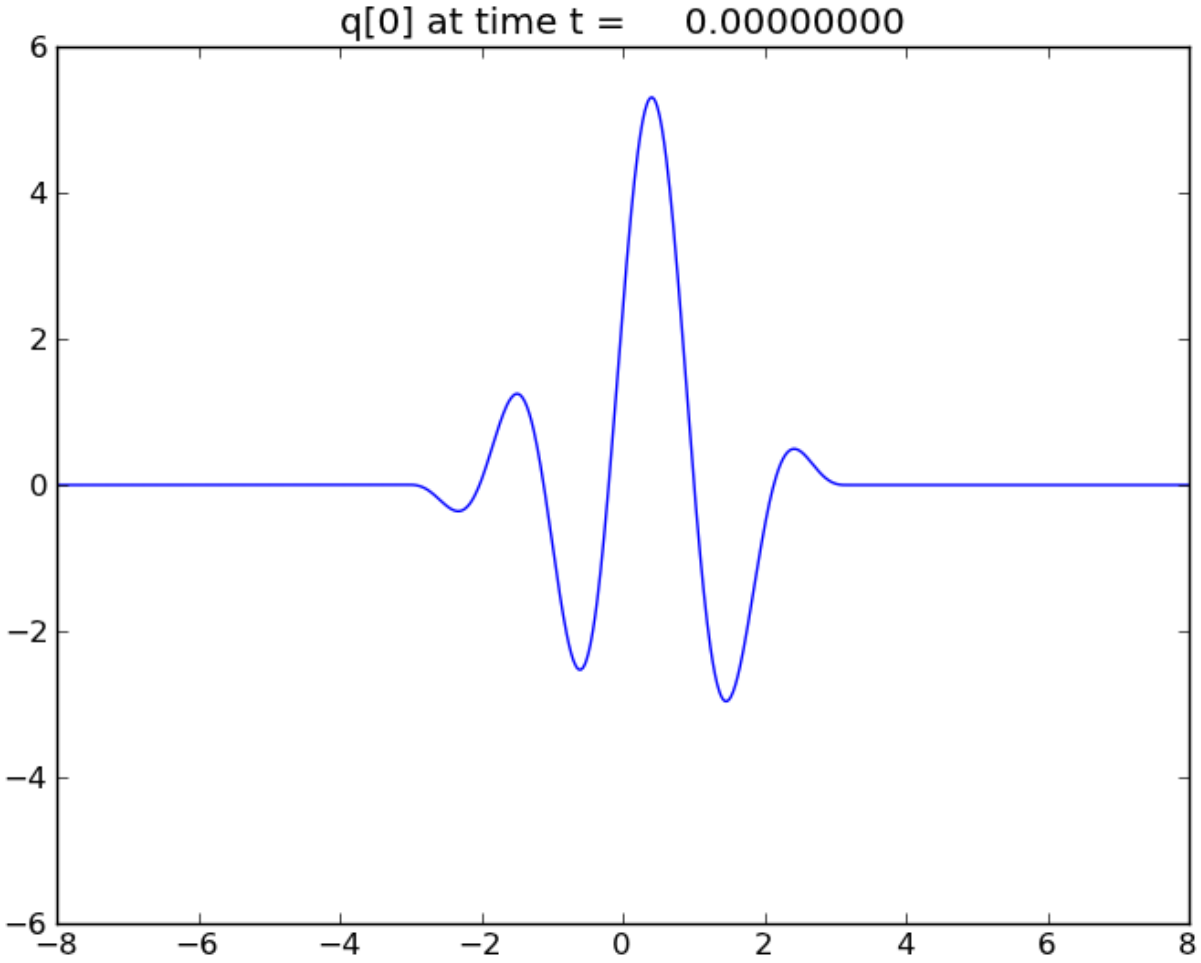


Viscous Burgers' Equation

- Now include a viscosity term: $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \varepsilon\frac{\partial^2 u}{\partial x^2}$
- Same initial condition: $u(x,0) = 1 - \cos(x)$

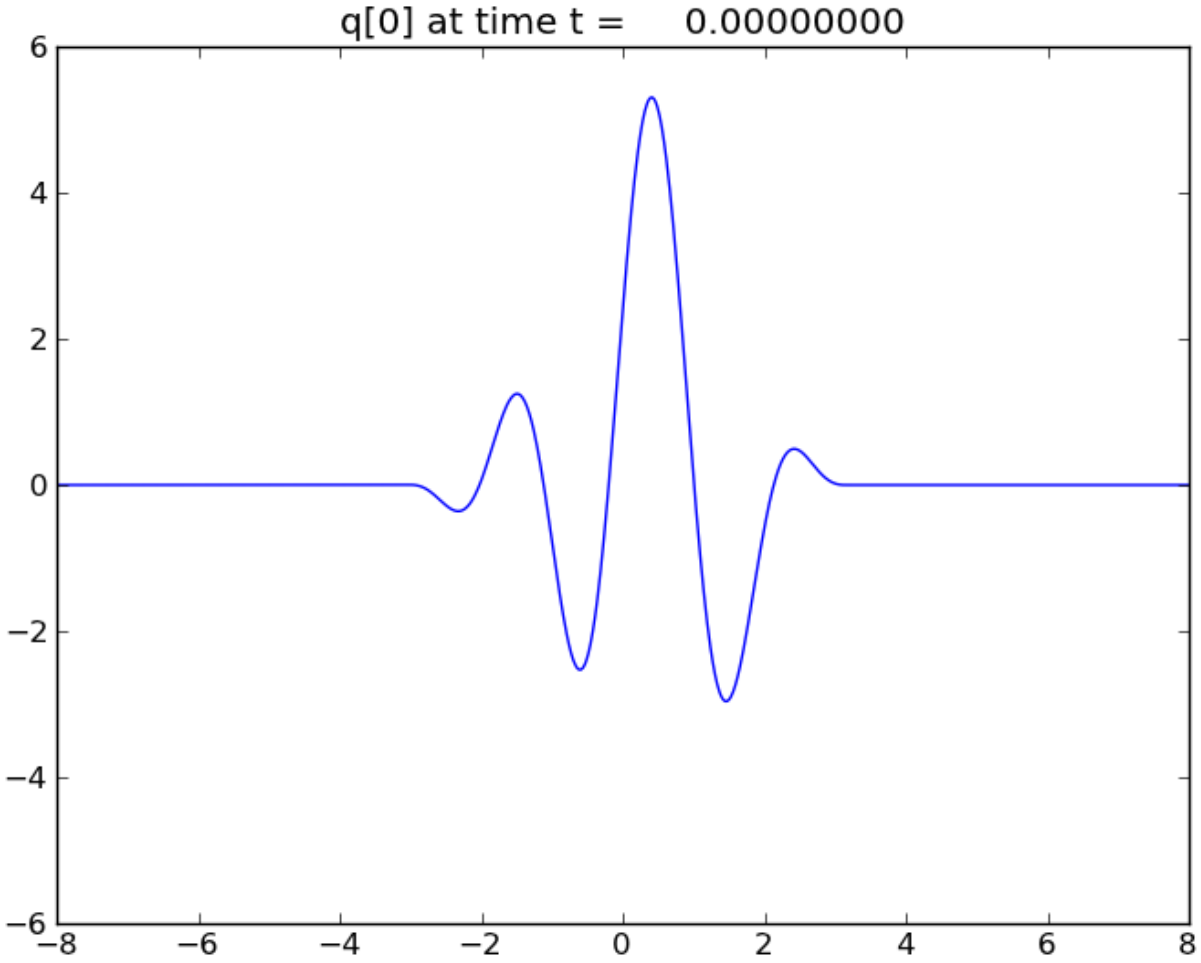


Example of shocks forming and merging



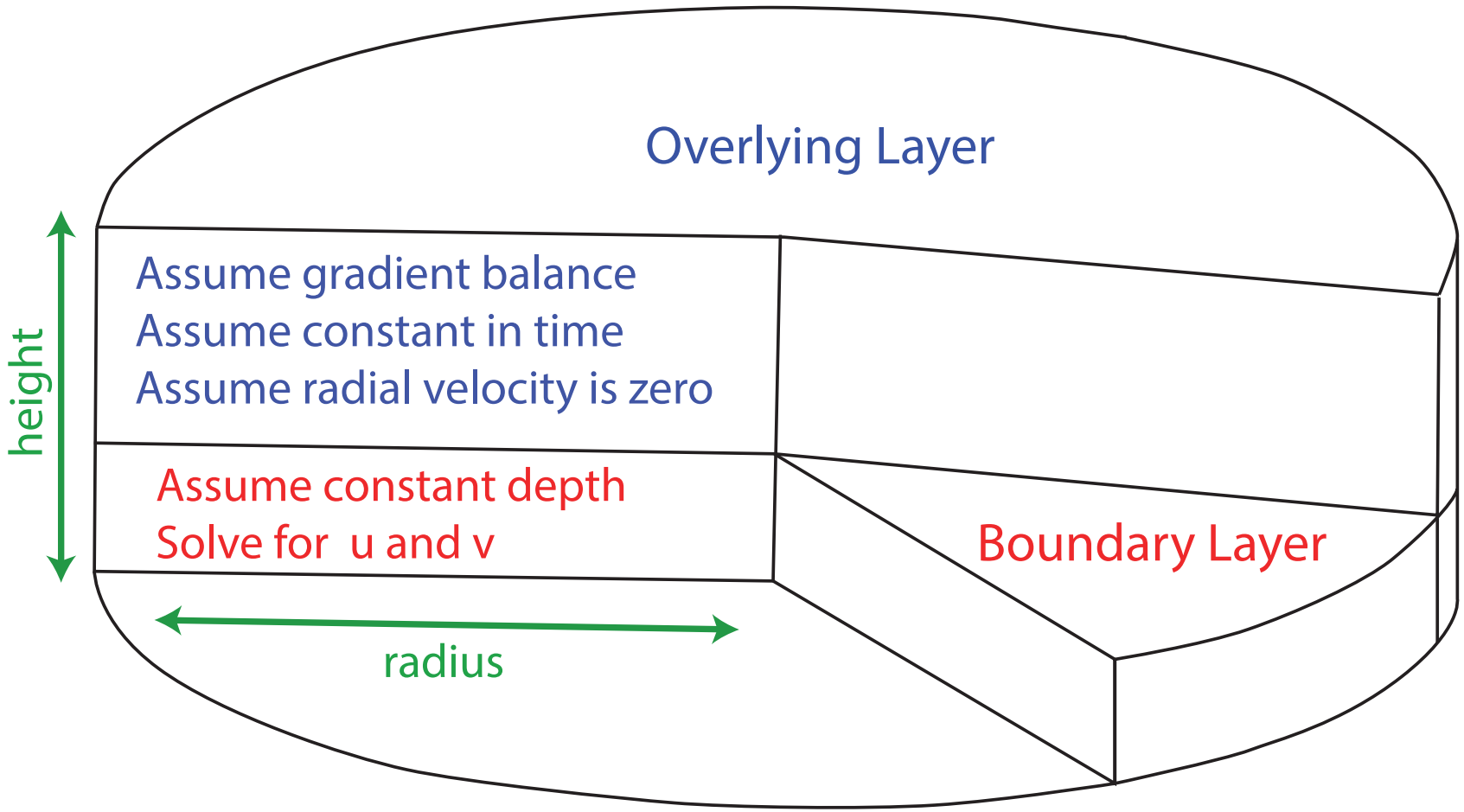
Based on LeVeque 2002

Example of shocks forming and merging



Based on LeVeque 2002

Slab Boundary Layer Model



Slab Boundary Layer Model - Governing Equations

Two predictive equations for the horizontal wind

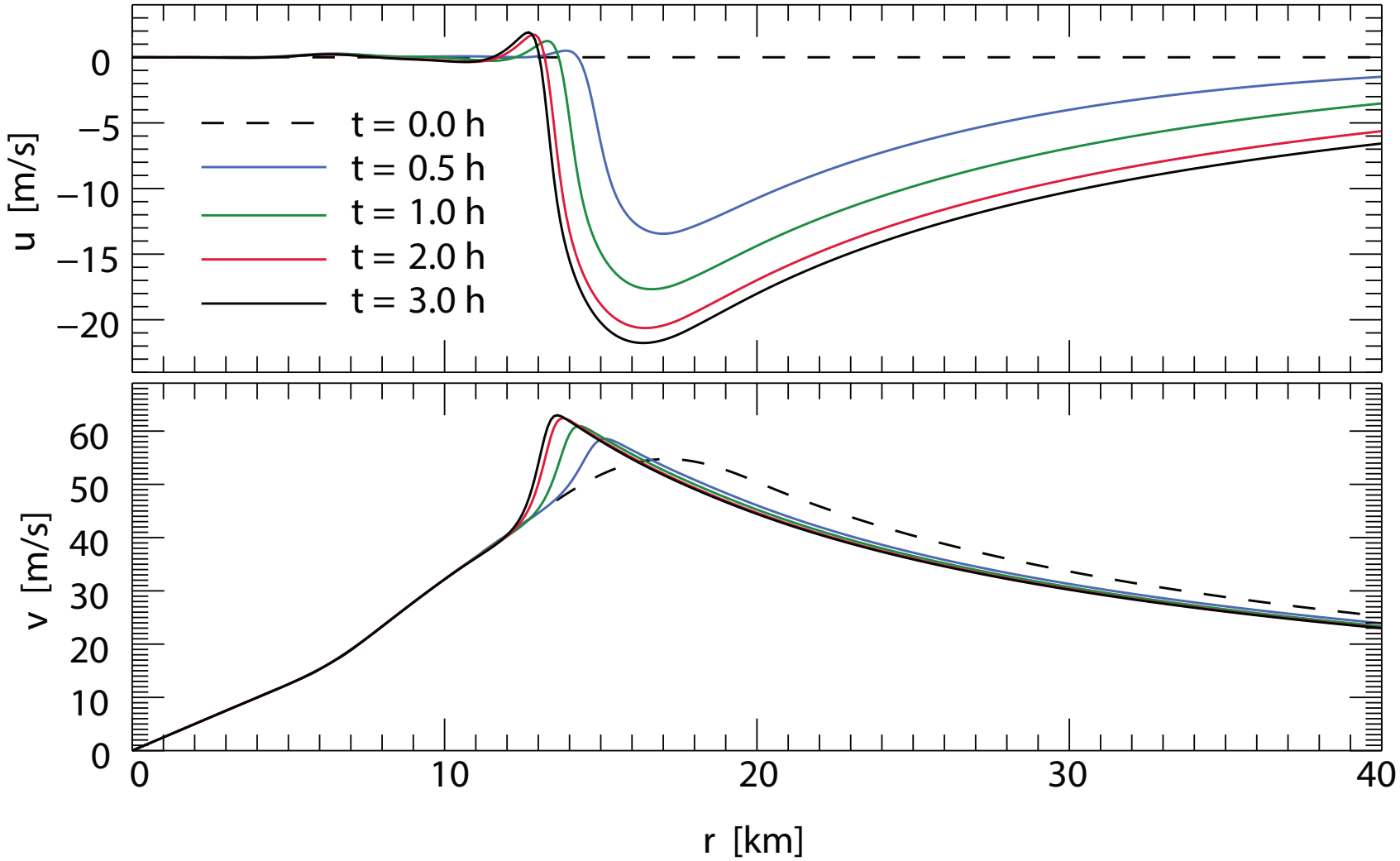
$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial r} - w^{-}\left(\frac{u}{h}\right) + \left(f + \frac{v + v_{\mathrm{gr}}}{r}\right)(v - v_{\mathrm{gr}}) - c_D U\frac{u}{h} + K\frac{\partial}{\partial r}\left(\frac{\partial(ru)}{r\partial r}\right)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial r} - w^{-}\left(\frac{v - v_{\mathrm{gr}}}{h}\right) - \left(f + \frac{\partial(rv)}{r\partial r}\right)u - c_D U\frac{v}{h} + K\frac{\partial}{\partial r}\left(\frac{\partial(rv)}{r\partial r}\right)$$

Two diagnostic equations for the vertical velocity

$$w = -h\frac{\partial(ru)}{r\partial r} \qquad \text{and} \qquad w^{-} = \frac{1}{2}(|w| - w)$$

Numerical results from Williams et al. 2013



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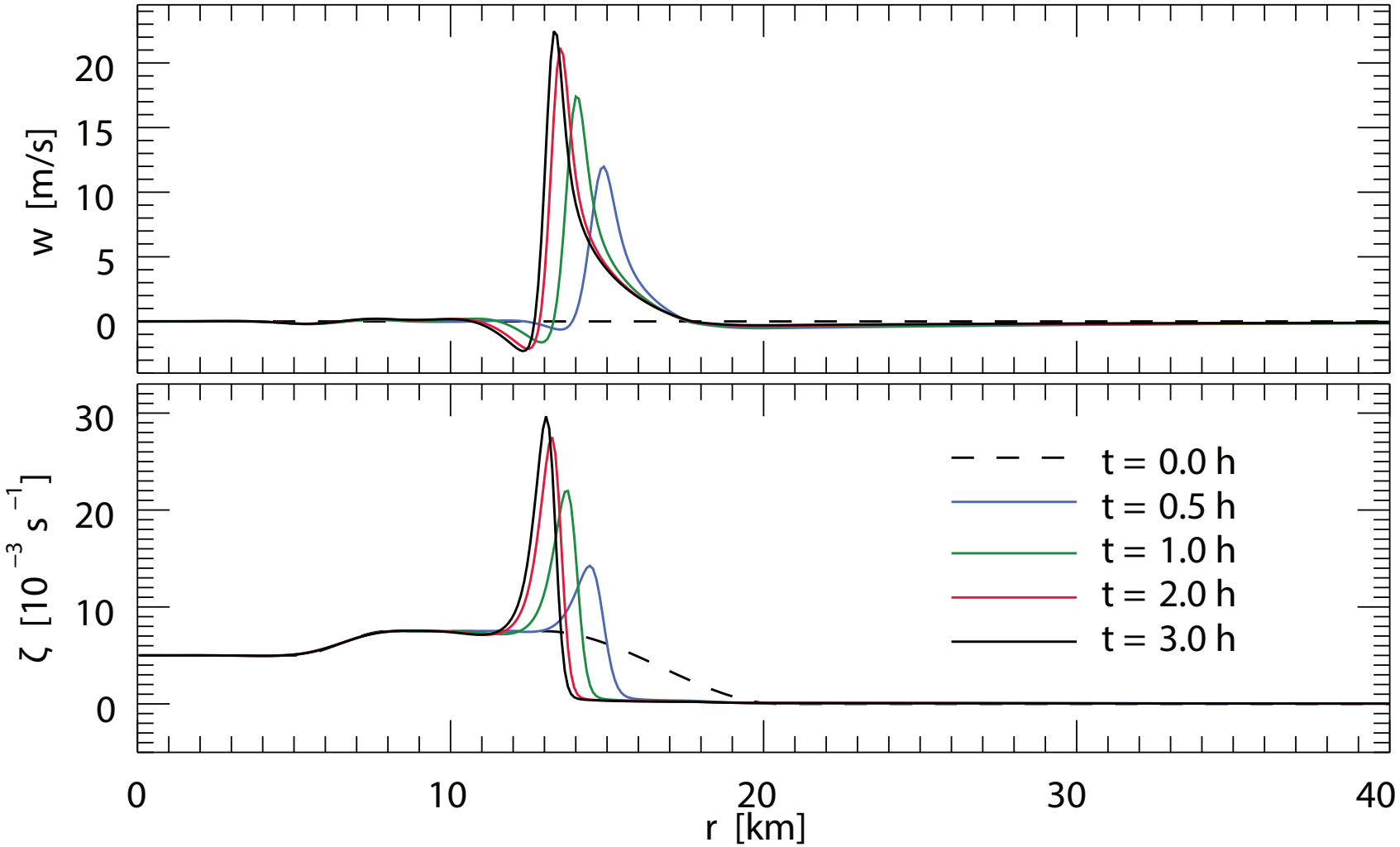


Simple Analytical Model - Equations

Through simplifying the governing equations

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial r} - \frac{u}{\tau}$$

$$\frac{\partial v}{\partial t} = -u\left(f + \frac{\partial v}{\partial r} + \frac{v}{r}\right) - \frac{v}{\tau}$$

where $\tau =$ a "typical" value of $h/(c_D U)$

Analytical solutions

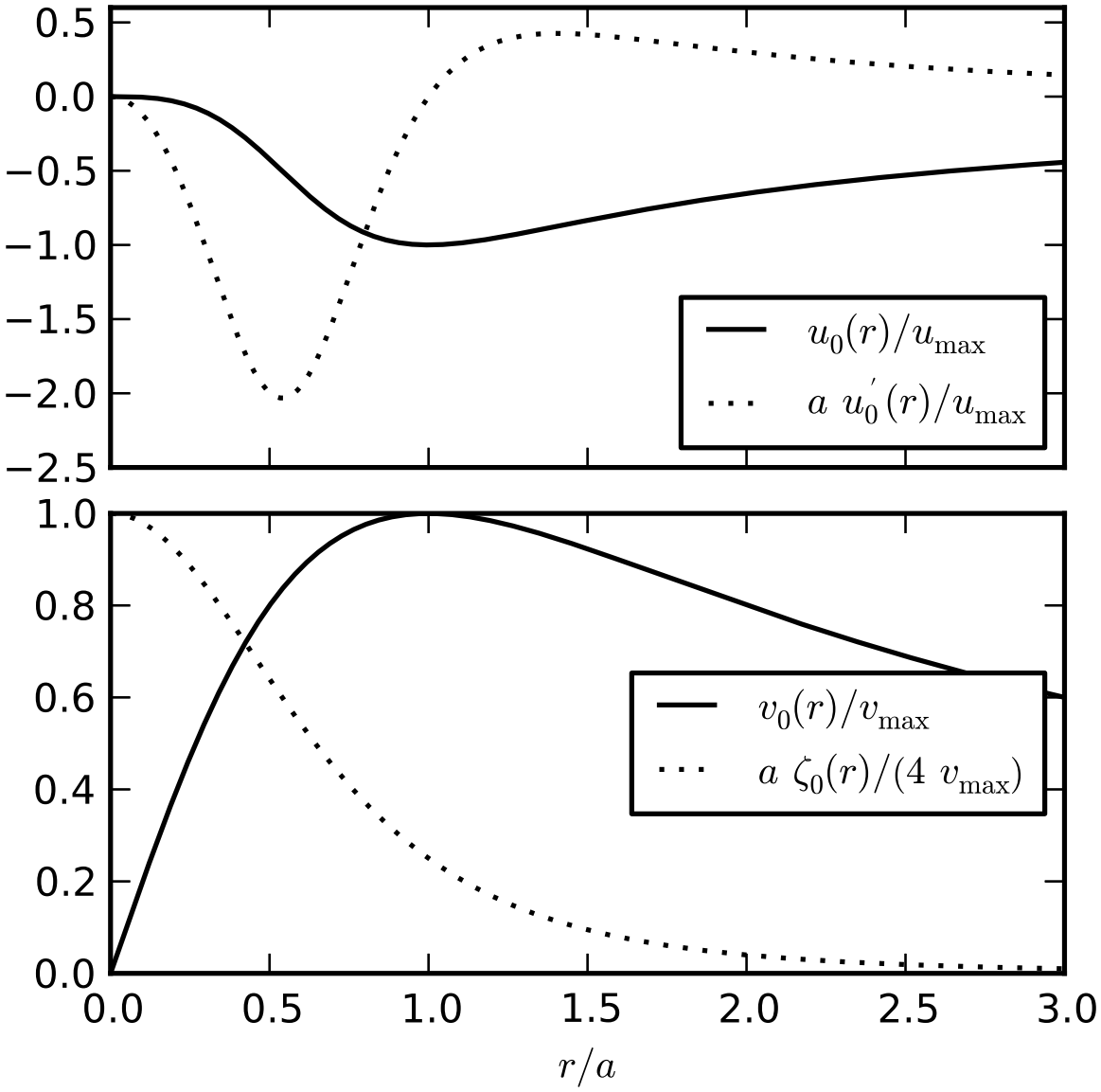
$$u(r,t) = u_{\mathrm{o}}(\hat{r})e^{-t/\tau}$$

$$rv(r,t) = \hat{r}v_{\mathrm{o}}(\hat{r})e^{-t/\tau}$$

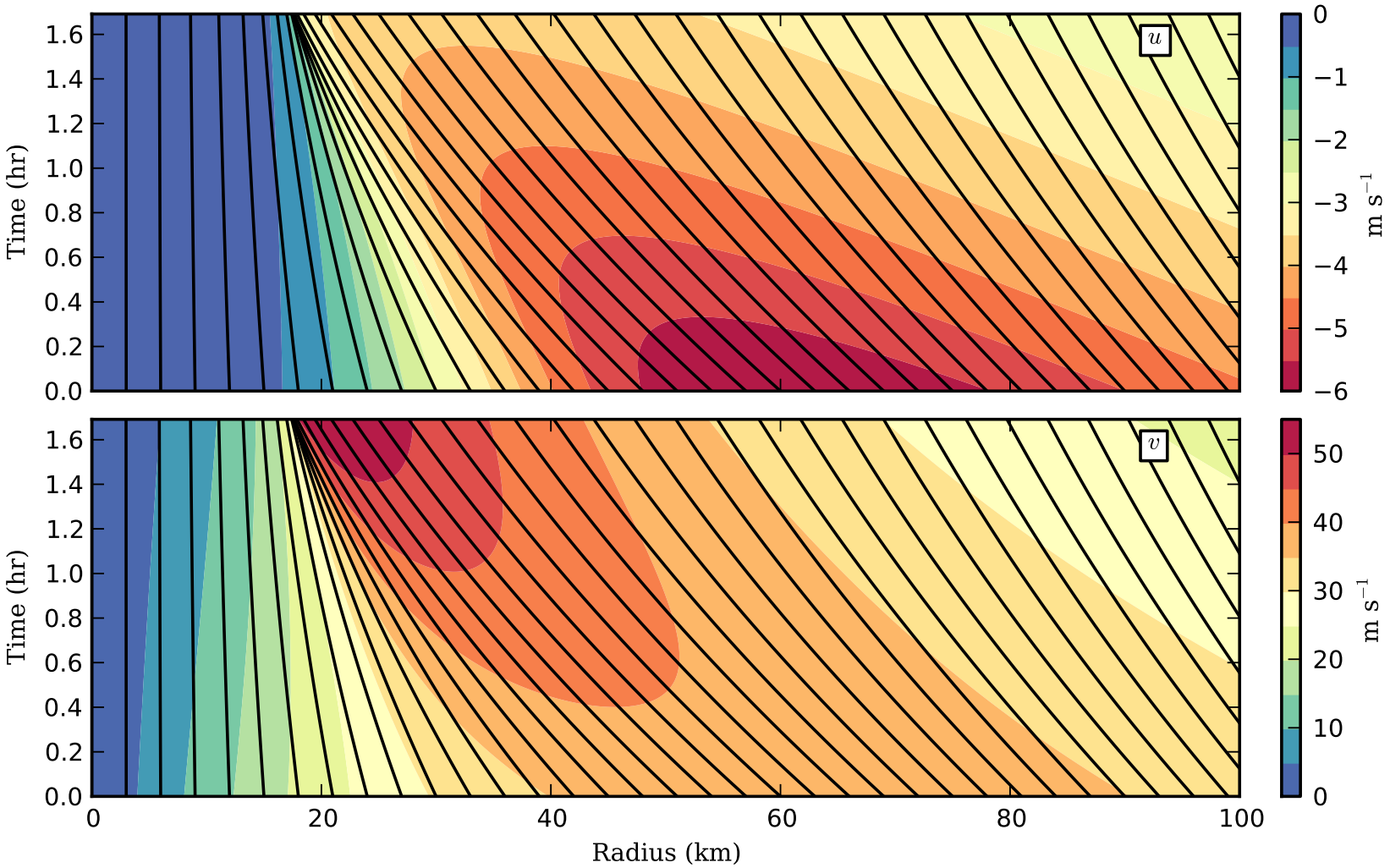
Characteristic equation

$$r = \hat{r} + \tau\left(1 - e^{-t/\tau}\right)u_{\mathrm{o}}(\hat{r})$$

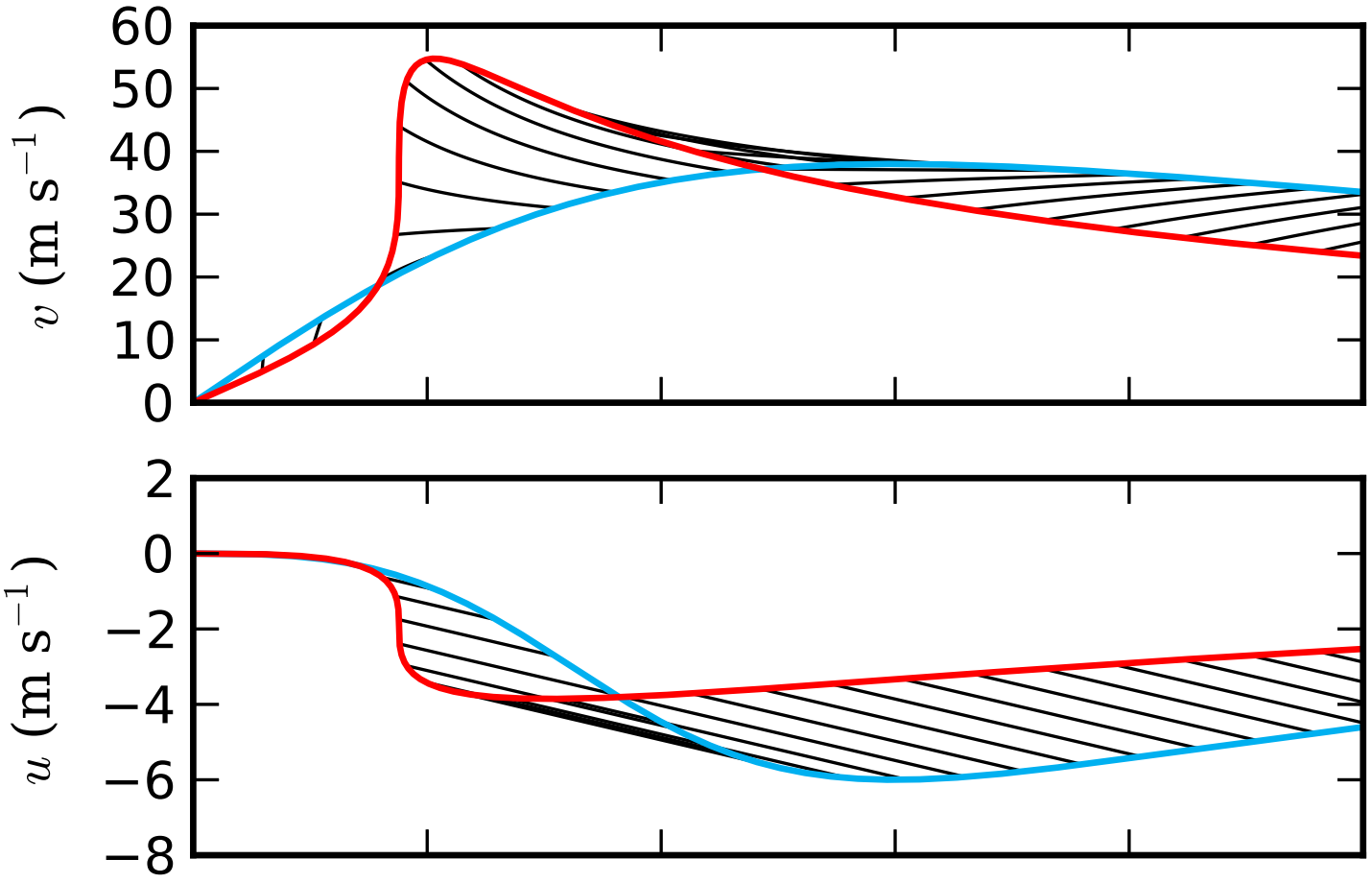
Initial conditions - Single eyewall



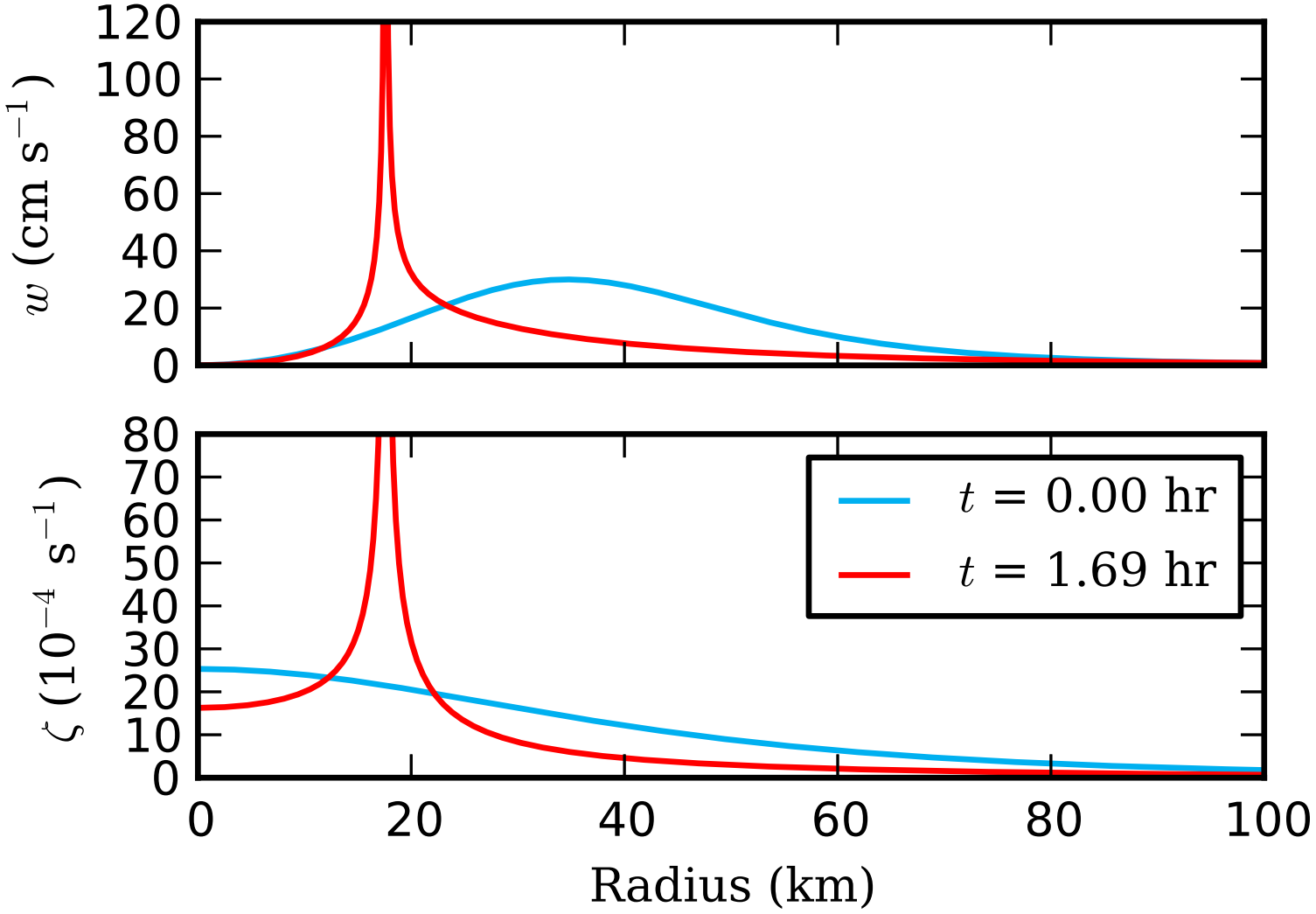
Results - Single eyewall



Results - Single eyewall



Results - Single eyewall

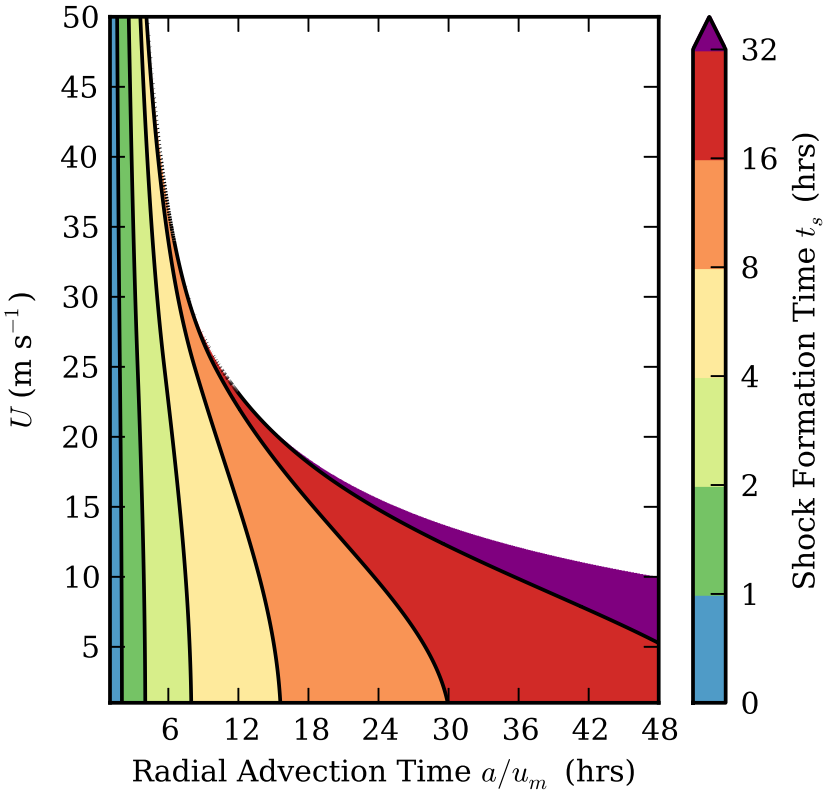


Time of shock formation

We can solve for the time of shock formation through

$$\text{Time of shock formation} = t_s = -\tau \ln\left(1 + \frac{1}{\tau u_{\mathrm{o}}'(\hat{r}_s)}\right)$$

$$\text{Radius of shock formation} = r_s = \hat{r}_s - \frac{u_{\mathrm{o}}(\hat{r}_s)}{u_{\mathrm{o}}'(\hat{r}_s)}$$

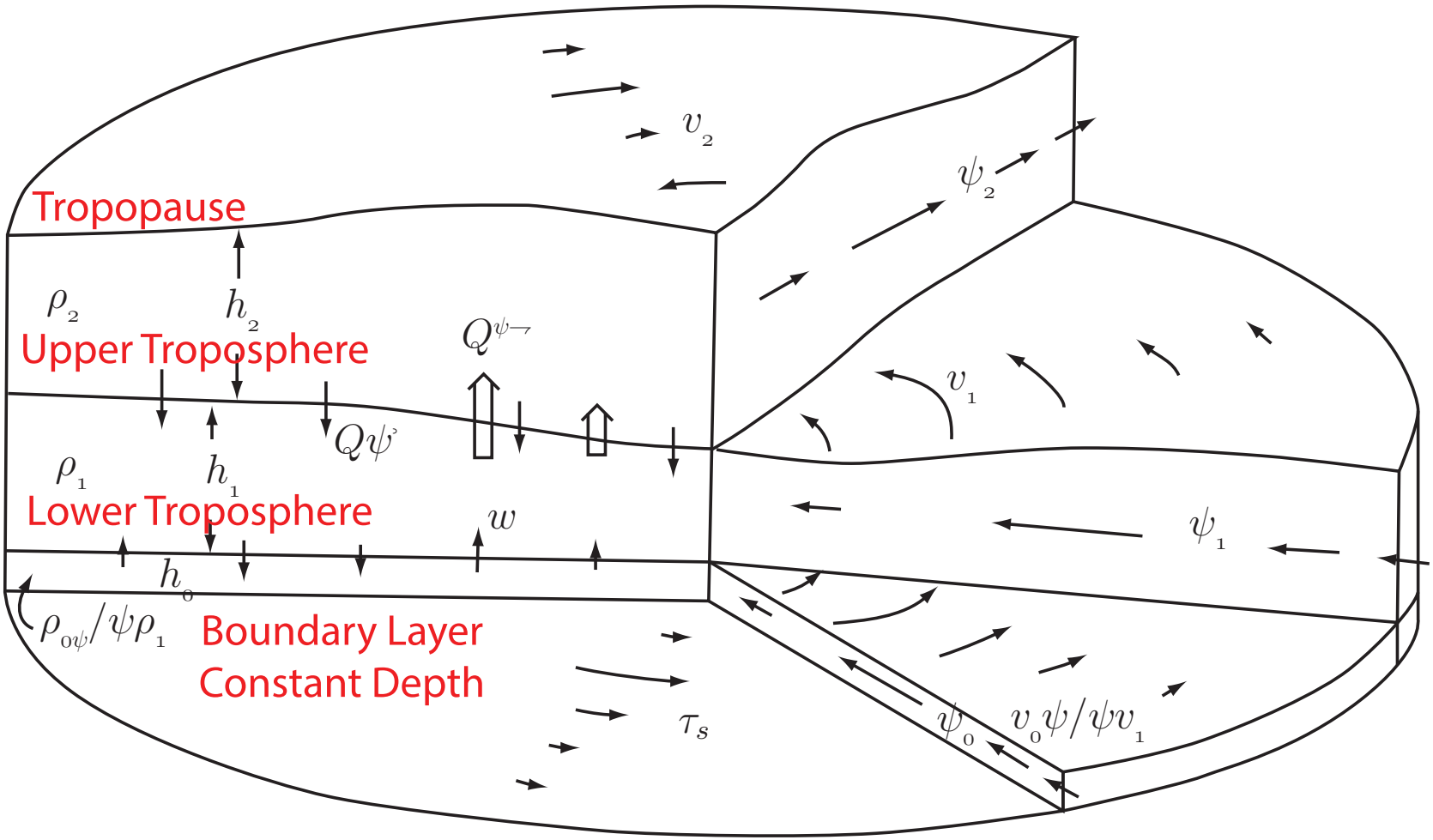


Conclusions

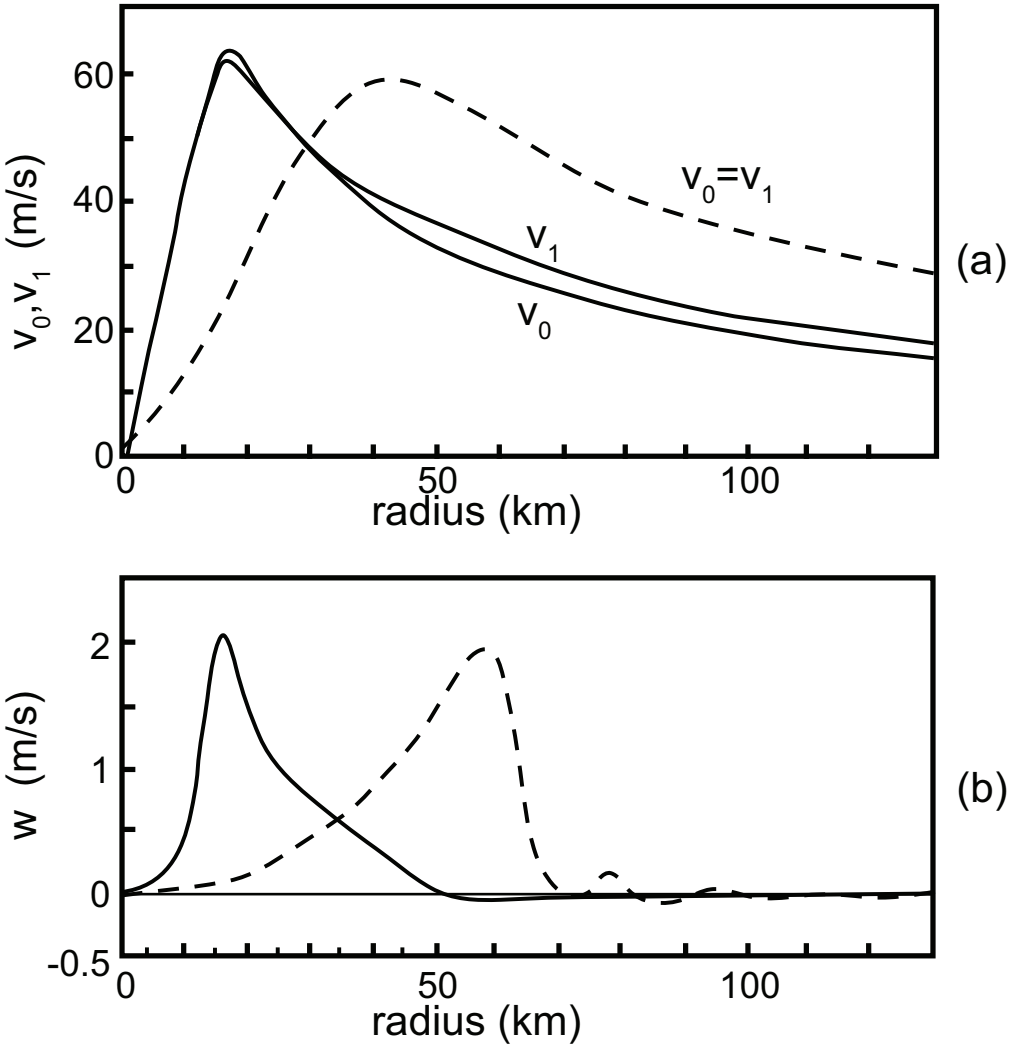
- 1-D solutions give a conceptual model of how diabatic heating influences intensity
- 1-D solutions inadequate at reproducing changes in modeled and observed storms
- 2-D solutions seem to have a more realistic structure and could be applied to modeled and observed storms
- 2-D solutions still neglect feedbacks from boundary layer
- Shock-like structures capture features in Hurricane Hugo (1989)
- Shocks control the size of the hurricane eye and the location of maximum Ekman pumping

Plans for creating a balanced model which includes frictional effects

Ooyama 1969a,b model design



Ooyama 1969a,b differences



Questions?